Math 2534  Methods of Proofs 2

Instructions: Either provide a proof or a counterexample for each of the following.

1) For all positive integers \( n \), if \( n \) is prime then \( n \) is odd. For \( n > 2 \)

2) For all integers \( a, b, \) and \( c \), if \( a \mid b \) and \( a \mid c \), then \( a \mid (b + c) \).

3) For all integers \( a, b, \) and \( c \), if \( a \) does not divide \( bc \) then \( a \) does not divide \( b \)
   Prove by contrapositive.

4) An integer \( n \) is even if and only if \( n^2 \) is even.

5) Prove that \( \sqrt{2} \) is irrational.

6) Given that \( n, a, b \) are integers. If \( n \mid a \) and \( n \mid (a + b) \), Then \( n \mid b \).

7) Suppose \( a \) and \( b \) are integers. The product \( ab \) is odd if and only if, \( a \) and \( b \)
   are both odd.

8) If \( ab \mid c \), Then \( a \mid c \) and \( b \mid c \). for integers \( a, b, c \)

9) If \( a \mid b \) and \( c \) then \( a \mid b \) for integers \( a, b, c \)

10) Given an integer \( n \), \( n \) is even iff \( 7n + 4 \) is even.

11) If \( a, b, c \) are prime numbers greater than 2, then \( a^3 + b^3 \neq c^3 \)

12) The sum of a rational and irrational number is irrational.