**Problem 1:**

Solve the initial value first order linear differential equations below. Indicate the largest interval that will guarantee a unique solution for the given initial value.

a) $ty' + y = t \cos(t), \quad y(\pi / 2) = 4$

Solution: $y' + \frac{1}{t} y = \cos(t)$ Largest interval to guarantee a unique solution for this initial is $(0, \infty)$. The integrating factor is $u(t) = e^{\int \frac{1}{t} dt} = e^{\ln(t)} = t$ which gives $[yt]' = t \cos(t)$. Now integrate to get $\int [yt]' dt = \int t \cos(t) dt$ to get $yt = \int t \cos(t) dt$ and $yt = t \sin(t) + \cos(t) + k$, so the general solution is $y = \sin(t) + \frac{\cos(t)}{t} + \frac{k}{t}$. Using $y(\pi / 2) = 4$, the particular solution is $y = \sin(t) + \frac{3\pi}{2t}$.

b) $(t+1)y' + y = 6, \quad y(1) = 1$

Solution: $y' + \frac{1}{t+1} y = \frac{6}{t+1}$, Largest interval for this initial value is $(-1, \infty)$

integrating factor $u(t) = e^{\int \frac{1}{t+1} dt} = e^{\ln(t+1)} = t+1$, so $[y(t+1)]' = 6$ and $\int [y(t+1)]' dt = \int 6 dt$ to get that $y(t+1) = 6t + k$. Solution is $y = \frac{6t+k}{t+1}$ and the particular solution for $y(1) = 1$ is $y = \frac{6t}{t+1} - 2$.

c) $y' + \sin(t)y = 0, \quad y(\pi / 2) = 2$

Solution: Largest interval for this initial value is $(-\infty, \infty)$

The integrating factor is $u(t) = e^{\int \sin(t) dt} = e^{-\cos(t)}$

The general solution is $y = Ce^{-\cos(t)}$

The particular solution for the initial value $y(\pi / 2) = 2$ is $y = 2e^{-\cos(t)}$. 


Problem 2:
Initially a tank contains 30 pounds of salt dissolved in 180 gallons of fresh water. A solution containing 1/3 lb of salt per gallon is added to the tank at a rate of 6 gal/min. At the same time a “well stirred” solution is leaving the tank at the same rate. Find the equation for the amount of salt in the tank at any time t. Determine the limiting factor for this solution.

Let Q be the amount of salt at any time t. The initial value is Q(0) = 30

\[
Q' = (1/3)(6) - (Q/180)(6)
\]

\[
Q' = 2 - \frac{Q}{30}
\]

\[
Q' + \frac{Q}{30} = 2 \quad \text{let } u(t) = e^{\int \frac{1}{30}dt} = e^{\frac{t}{30}}
\]

\[
[Qe^{\frac{t}{30}}]' = 2e^{\frac{t}{30}}, \quad \text{Now integrate to get } Qe^{\frac{t}{30}} = 60e^{\frac{t}{30}} + k
\]

and \( Q = 60 + ke^{\frac{-t}{30}} \). So the particular solution is \( Q = 60 -30e^{\frac{-t}{30}} \)

and \( \lim_{t \to \infty} Q = 60 \) Therefore in the most amount of salt possible is 60 lbs.