Math 2214: Solution Homework 1 Spring 2016 sec 1.1-2.1

1) You are given that \( y(t) = 2e^{-4t} \) is a solution for the differential equation \( y' + ky = 0 \) with the initial value (boundary condition) \( y(0) = y_0 \). Solve for \( k \) and \( y_0 \) so that this solution is valid.

Solution:

\[ y(t) = 2e^{-4t} \]
\[ y'(t) = -8e^{-4t} \]

Now substitute into \( y' + ky = 0 \)

\[ y' + ky = 0 \]
\[ -8e^{-4t} + 2ke^{-4t} = 0 \]

\( e^{-4t}[2k - 8] = 0 \) and \( e^{-4t} \neq 0 \) so \( 2k - 8 = 0 \) so \( k = 4 \)

Using the initial value \( y(0) = y_0 \) and the solution \( y(t) = 2e^{-4t} \) we have \( y_0 = 2e^0 \) so \( y_0 = 2 \).

2) Using the method of isoclines, sketch the direction field filaments (not the solutions) for the following: (Do about 4 different Isoclines with filaments for each DE)

a) \( y' = t^2 + y \) for \( t \geq 0, y \geq 0 \)

Let the slope \( y' = C \) so that \( C = t^2 + y \) and \( y = C - t^2 \) is the isocline for some value of \( C \). Let \( C = 0, C = 1, C = 3, C = 5 \)

![Isocline Diagram](image)

b) \( y' = ty \) for \( t \geq 0, y \geq 0 \)

Let the slope \( y' = C \) so that \( C = ty \) and \( y = C/t \) is the isocline for some value of \( C \). Let \( C = 0, C = 1, C = 3, C = 5 \)

![Isocline Diagram](image)
3) The following DE is autonomous. Find the equilibrium isoclines and sketch them with filaments (Do not try to sketch the actual solutions for problem below)

\[ y' = y^2 + 8y + 12 \quad \text{over } (-\infty, \infty) \]
Let the slope \( y' = C \) and let \( C = 0 \).
We now have \( 0 = y^2 + 8y + 12 = (y+6)(y+2) \), so \( y = -6 \) and \( y = -2 \) are the equilibrium solutions.

\[ \begin{array}{c|c}
\text{t} & 0 \\
\hline
\text{y} & -2 \\
\hline
\end{array} \]

4) Which of the following differential equations are linear or non-linear?
Put the linear differential equations in proper linear form.

a) \[ \frac{dy}{dt} = e^{2t} - y \] Linear \( y' + y = e^{2t} \)

b) \[ \frac{dy}{dt} = \frac{y}{t} \] Linear \( ty' - y = 0 \)

c) \[ ty' - \ln y = t^2y \] Non - Linear since \( ty' - t^2y = \ln |y| \) and \( \ln |y| \neq g(t) \)

d) \[ y'' + e^t y' = t^2 y^2 \] Non-linear since exponent greater than 1.

e) \[ y^{(3)} + \sin(t)y' = t \] Non-linear since there is a product of the solution and its derivative.
5) Determine the DE that is represented by this direction field.
   a) \( y' = (-y - 2)(y - 1) \)
   b) \( y' = (y - 1)(y + 2) \)

Since both Diff Eqs have the same equilibrium solutions, \( y = 1 \) and \( y = -2 \), we need to

test other isoclines. For instance test \( y = 4 \).

a) \( y' = (-y - 2)(y - 1) = (-6)(3) = -18 \)

b) \( y' = (y - 1)(y + 2) = (3)(6) = 18 \)

Equation a) satisfies this test since \( y = 4 \) has negative slopes at each point and in
equation b) has positive slopes at each point on \( y = 4 \).