1) a) Verify that \( y = c_1e^{-t} + c_2e^{-4t} \) is a solution for \( y'' + 5y' + 4y = 0 \)?
\( c_1 \) and \( c_2 \) are arbitrary unknown real numbers.

Solution:
Consider \( y = c_1e^{-t} + c_2e^{-4t} \)
\[
\begin{align*}
y' &= -c_1e^{-t} - 4c_2e^{-4t} \\
y'' &= c_1e^{-t} + 16c_2e^{-4t}
\end{align*}
\]
Now substitute in the DE to show it is a solution.
\[
y'' + 5y' + 4y = 0 \\
(c_1e^{-t} + 16c_2e^{-4t}) + 5(−c_1e^{-t} − 4c_2e^{-4t}) + 4(c_1e^{-t} + c_2e^{-4t}) = \\
c_1e^{-t} + 16c_2e^{-4t} - 5c_1e^{-t} - 20c_2e^{-4t} + 4c_1e^{-t} + 4c_2e^{-4t} = 0
\]

b) If \( y(0) = 4 \) and \( y'(0) = -1 \), Solve for \( c_1 \) and \( c_2 \).
If \( y = c_1e^{-t} + c_2e^{-4t} \) and \( y(0) = 4 \), then \( 4 = c_1 + c_2 \)
If \( y' = -c_1e^{-t} - 4c_2e^{-4t} \) and \( y'(0) = -1 \), then \(-1 = -c_1 - 4c_2 \)
Solving for \( c_1 \) and \( c_2 \), we have that \( c_1 = 5 \) and \( c_2 = -1 \)

2) For what value of “\( k \)” is \( y(t) = e^{kt} \) a solution of \( 2y'' - 4y = 0 \).
Suppose that \( y = e^{kt} \) is a solution for \( 2y'' - 4y = 0 \).
Then \( y' = ke^{kt} \) and \( y'' = k^2e^{kt} \) and \( 2(k^2e^{kt}) - 4(e^{kt}) = 0 \)
Since \( e^{kt} \) is never equal to zero then \( 2k^2 - 4 = 0 \) so \( k = \pm \sqrt{2} \)

3) For each problem a) and b), find and sketch two isoclines for the slope values \( C = 1 \) and \( C = 5 \) in first quad only. Problem c) is autonomous. Find the equilibrium isoclines and sketch them. (Do not try to sketch the actual solutions for problems below)

a) \( y' = -t^2 + y \) for \( t \geq 0, y \geq 0 \)
Solution: The general isocline is \( C + t^2 = y \).
The two isoclines to sketch are \( 1 + t^2 = y \) (the slope at every point on this isocline is 1) and \( C + t^2 = y \) (the slope at every point on this isocline is 5).
b) \( y' = ty \) for \( t \geq 0, y \geq 0 \)

Solution: The general isocline is \( \frac{C}{t} = y \)

The two isoclines to sketch are \( \frac{1}{t} = y \) (the slope at every point on this isocline is 1) and \( \frac{5}{t} = y \) (the slope at every point on this isocline is 5).

c) \( y' = y^3 - 5y^2 + 6y \) for \( t \geq 0, y \geq 0 \)

This DE is autonomous. To find the equilibrium isoclines, set the slope \( y' \) equal to zero so that \( 0 = y^3 - 5y^2 + 6y = y(y - 3)(y - 2) \), and \( y = 0, y = 3 \) and \( y = 2 \). Each of these isoclines is horizontal and an equilibrium isocline. The slope at each point on each of these isoclines is zero.

4) Which of the following differential equations are linear?

   Put the linear differential equations in proper linear form.

   a) \( \frac{dy}{dt} = y \tan(t) + y \cot(t) \)

   Linear: \( y' - y(\tan(t) + \cot(t)) = 0 \)

   b) \( \frac{dy}{dt} = \frac{y^2}{t} \)

   Non-Linear since the term \( y^2 \) is included in the equation.

   c) \( t^2 \frac{dy}{dt} - \sin(t) = \frac{y^{(2)}}{t} \)

   Linear: \( -\frac{y^{(2)}}{t} + t^2 y = \sin(t) \) or \( -y^{(2)} + ty = \frac{\sin(t)}{t} \)

   d) \( \frac{dy}{dt} + y \ln(t) - e^t \frac{d^3 y^3}{dt^3} = 0 \)

   Non Linear: Since \( \frac{dy}{dt} + y \ln(t) - e^t [6y' + 12yy'' + 3y^2 y'''] = 0 \)

   e) \( y'' + y' \sin(y) - \frac{y}{t} = 0 \)

   Non-Linear since the term \( \sin(y) \) is included in the equation.