Math 2214: Homework 1 (sec 1.1-1.3) Spring 2015

1) a) Verify that \( y = c_1 e^{-t} + c_2 e^{4t} \) is a solution for \( y'' + 5y' + 4y = 0 \)? (\( c_1 \) and \( c_2 \) are arbitrary unknown real numbers.)

b) If \( y(0) = 4 \) and \( y'(0) = -1 \), Solve for \( c_1 \) and \( c_2 \).

2) For what value of “\( k \)" is \( y(t) = e^{kt} \) a solution of \( 2y'' - 4y = 0 \).

3) For each problem a) and b), find and sketch two isoclines for the slope values \( C = 1 \) and \( C = 5 \) in first quad only. Problem c) is autonomous. Find the equilibrium isoclines and sketch them. (Do not try to sketch the actual solutions for problems below)

a) \( y' = -t^2 + y \) for \( t \geq 0, y \geq 0 \)

b) \( y' = ty \) for \( t \geq 0, y \geq 0 \)

c) \( y' = y^3 - 5y^2 + 6y \) for \( t \geq 0, y \geq 0 \)

4) Which of the following differential equations are linear? Put the linear differential equations in proper linear form.

a) \( \frac{dy}{dt} = y \tan(t) + y \cot(t) \)

b) \( \frac{dy}{dt} = \frac{y^2}{t} \)

c) \( t^2 \frac{dy}{dt} - \sin(t) = \frac{y^{(3)}}{t} \)

d) \( \frac{dy}{dt} + y \ln(t) - e^t \frac{d^3 y^3}{dt^3} = 0 \)

e) \( y'' + y' \sin(y) - \frac{y}{t} = 0 \)