Review Practice Problems for Math 1526 Final Exam

1) Given the following matrices, find:
   a) \( 7A - 6B \)    b) \( BC \)
   c) \( A^T \)    d) \( C^{-1} \)
   e) If \( IB = B \) what size is \( I \)?

\[
A = \begin{bmatrix}
1 & 6 \\
4 & 0 \\
2 & 1 \\
-3 & -2
\end{bmatrix} \quad \quad B = \begin{bmatrix}
1 & 3 \\
0 & -1 \\
-2 & 1 \\
3 & -5
\end{bmatrix} \quad \quad C = \begin{bmatrix}
1 & 0 & 3 \\
-1 & 2 & 1
\end{bmatrix}
\]

2) An important part of an archaeologist's work is to assign dates to deposits and finds. Below is a matrix which records this information. The pottery finds from a dig have been divided into 4 stratified deposits and sorted into six types of pottery. The 4 X 6 matrix is defined by \( b_{ij} = 1 \) if the deposit \( i \) contains pottery type \( j \) and \( b_{ij} = 0 \) if not.

\[
B = \begin{bmatrix}
0 & 0 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

   a) Explain the meaning of entry \( b_{35} \)

3) Two Types of Juices A and B (represented respectively by the rows) are blended from four other juices: grape, cranberry, tangerine, and strawberry (represented respectively by the columns). The number of gallons of each ingredient in a 20- gal container of each juice A and B are given by the matrix \( J \).

\[
J = \begin{bmatrix}
8 & 6 & 4 & 2 \\
9 & 7 & 3 & 1
\end{bmatrix}
\]

   a) Set up a cost matrix \( C \) if grape juice costs $1 a gal, cranberry cost $2 a gal, tangerine cost $1 a gal and strawberry cost $3 a gal.

   b) Find the total cost of making 20 gallons of each juice A and B using matrix multiplication. (Answer in a sentence).

\[
\begin{bmatrix}
1 & 0 & -1 & 0 \\
0 & 1 & 3 & 1 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 1
\end{bmatrix} \quad \begin{bmatrix}
2 \\
0 \\
2 \\
1
\end{bmatrix}
\]

4) Use \( A^{-1} \) to solve the system \( AX = B \) when \( A = \begin{bmatrix}
1 & 0 & -1 & 0 \\
0 & 1 & 3 & 1 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 1
\end{bmatrix} \) and \( B = \begin{bmatrix}
2 \\
0 \\
2 \\
1
\end{bmatrix} \).
5) A botanist can purchase plant food of 4 different I, II, III, and IV that come in the same size bag. He needs to decide how many bags of each type plant food he needs to purchase to obtain the required amount of nutrients A, B and C. Use the table to set up your system of equations where x, y, z, and w represent the number of bags of Type I, II, III, and IV respectively. Solve using the Gauss-Jordan method of reduction. Put the augmented matrix into reduced row echelon form. You will get the following infinite solution: \((3w, 1000 - 2w, 500 - w, w)\). State the restriction on \(w\).

<table>
<thead>
<tr>
<th>Food</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>Total requirement of nutrients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nutrient A</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>5</td>
<td>10000</td>
</tr>
<tr>
<td>Nutrient B</td>
<td>10</td>
<td>5</td>
<td>30</td>
<td>10</td>
<td>20000</td>
</tr>
<tr>
<td>Nutrient C</td>
<td>5</td>
<td>15</td>
<td>10</td>
<td>25</td>
<td>20000</td>
</tr>
</tbody>
</table>

6) The amount of carbon in the atmosphere increases at a rate of 43% per year. This increase is due to fossil-fuel emissions. The rate of increase is modeled by the differential equation \( \frac{dE}{dt} = 0.43E \) where \( E \) is the fossil-fuel emissions in gigatons per year and \( t \) is the time in years. In 1860 the fossil fuel emissions was 0.1. Predict the emissions for the year 2001. [Solve the differential equation to get the exponential growth equation]

7) The Utility function \( U(x,y) \) measures the customer satisfaction derived by consumption of two goods \( x \) and \( y \). Suppose that the utility function for two commodities is given by \( U(x,y) = xy^2 \) with a budget constraint given by \( x + y = 24 \). What value of \( x \) and \( y \) will maximize utility? (Use Lagrange Method.)

8) The profit for a grain crop is related to fertilizer and labor. The Profit per acre is given by \( P(x,y) = 100x + 40y -5x^2 - 2y^2 \) where \( x \) is the number of units of fertilizer and \( y \) is the number of work hours.
   a) What values of \( x \) and \( y \) will maximize profit? Do not forget to verify your conclusion.
   b) Evaluate \( \frac{\partial P}{\partial x} \) at \( x = 6 \) and \( y = 5 \) and explain the meaning of your answer.

9) Values of a linear cost function and a revenue function are given in the table.

<table>
<thead>
<tr>
<th>q</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(q)</td>
<td>500</td>
<td>700</td>
<td>900</td>
<td>1100</td>
<td>1300</td>
</tr>
<tr>
<td>R(q)</td>
<td>0</td>
<td>450</td>
<td>900</td>
<td>1350</td>
<td>1800</td>
</tr>
</tbody>
</table>

a) Write the cost equation, and indicate the fixed costs and the variable cost per unit.
b) What is the marginal cost and what does it mean?
c) Find the break-even point and explain what it means.
10) A monopoly has a cost function of \( C(x) = 1000 + 120x + 6x^2 \) for its product. The demand function is \( p = 360 - 3x - 2x^2 \).
   a) Find the revenue function \( R(x) \).
   b) Find the Profit equation \( P(x) \).
   c) Find the number of items that must be sold to earn the maximum profit.
   d) Set up (only) the integral to find the Consumer Surplus at the value \( x = 6 \).

11) Given that the marginal revenue function for a product is \( MR = 25 - q \) and the marginal cost function is \( MC = 4q \), find the profit equation \( P(q) \) if the daily fixed cost is $200.

12) An environmental group wishes to estimate the total area of algae growth in a pond. They measure the algae growth in sections across the pond and obtain the following data:

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>0</td>
<td>15</td>
<td>18</td>
<td>18</td>
<td>30</td>
<td>27</td>
<td>24</td>
<td>23</td>
<td>0</td>
</tr>
</tbody>
</table>

   a) Use the Trapezoidal Rule to approximate the area of the algae growth.

13) A scholarship fund is to provide an annual scholarship of $8000. If the annual rate of interest is 9% compounded continuously. How much should be invested to fund the scholarship indefinitely?

14) A store finds that it’s sales change at a daily rate given by \( s'(t) = -3t^2 + 300t \) where \( t \) is the number of days after an advertising campaign ends \( (0 \leq t \leq 30) \) Find the sales for the first seven days after the campaign ends.

5) Evaluate the following:

   a) \( \int_{0}^{2} 5x^3 \ (x^4 + 2)^3 \ dx \)

   b) \( \int_{1}^{e} e^{3x+2} \ dx \)

   c) \( \int_{-2}^{\infty} -2 / x^2 \ dx \)

**Warning (☺): You also need to review module examples, worksheets, old tests, test reviews as well as your textbook and matrix booklet.**