Problem 1: Given \( H(x) = \frac{x^2 + 3x - 10}{x - 2} \), define \( H(x) \) so that it’s continuity is extended to \( x = 2 \).

Since \( H(x) = \frac{(x+5)(x-2)}{(x-2)} \) we notice that \( H(x) \) has a hole at \( x = 2 \). The \( y \) value that \( H(x) \) is approaching as \( x \) approaches 2 is found as follows: \( \lim_{x \to 2} (x+5) = 7 \)

Now Define \( H(x) \) to be a continuous function:

\[
H(x) = \begin{cases} 
\frac{x^2 + 3x - 10}{x - 2} & \text{if } x \neq 2 \\
7 & \text{if } x = 2
\end{cases}
\]

Problem 2: Use the Intermediate Value Theorem to determine if the polynomial function \( f(x) \) has at least one root on the interval \([1, 3]\) when you know that \( f(1) = -5 \) and \( f(3) = 63 \). Explain and justify your reasoning.

Since we are given that \( f(x) \) is a polynomial is continuous and we are looking over the closed interval \([1, 3]\) we can us the IVT to determine if the there is a root for \( f(x) \) over this interval.

By the IVT we can choose a \( y_0 \) value between \( f(1) \) and \( f(5) \) and there will be a \( c \) value in \([1, 3]\) so that \( f(c) = y_0 \). In this case we need to choose \( y_0 = 0 \) and this is possible since we have that \( f(1) \) is a negative value and \( f(3) \) is a positive value. Therefore we have shown that \( f(x) \) does have at least one root over \([1, 3]\)