Math 1205 Solution Homework 4 Fall 2012

Evaluate the following limits:

1) Limit \( \lim_{x \to \infty} \frac{x + 4x^5}{2x^5 + x^4} = \lim_{x \to \infty} \frac{4x^5}{2x^5} = 2 \)

2) Limit \( \lim_{x \to -\infty} \frac{3x + 4x^5}{2x^3 + x^5} = \lim_{x \to -\infty} \frac{4x^5}{x^5} = 4 \)

3) Find the following limit and show all work.
\[
\lim_{x \to 0} \frac{3x}{5 \sin(2x)} = \frac{3}{5} \lim_{x \to 0} \frac{x}{\sin(2x)} = \frac{3}{10} \lim_{x \to 0} \frac{2x}{\sin(2x)} = \frac{3}{10} \lim_{x \to 0} \frac{1}{\frac{\sin(2x)}{2x}} = \frac{3}{10}
\]

4) Given the following information, sketch the graph that best fits the description
a) \( f(2) = 1, \ f(-1) = 0 \)

b) \( \lim_{x \to \infty} f(x) = 0 \) and \( \lim_{x \to -\infty} f(x) = 1 \)

c) \( \lim_{x \to 0^+} f(x) = \infty \) and \( \lim_{x \to 0^-} f(x) = -\infty \)

5) Use the formal definition for \( \lim_{x \to a} f(x) = \infty \) the verify that the following limit is valid

Given that \( \frac{1}{(x+3)^4} > 1000 \), Find \( \delta \)

\( \lim_{x \to a} \frac{1}{(x+3)^4} = \infty \)

Consider \( \frac{1}{(x+3)^4} > M \) and choose \( M = 1000 \)

\[
\frac{1}{(x+3)^4} > 1000
\]

\[
\frac{1}{1000} > (x+3)^4
\]

\[
\sqrt[4]{1/1000} > |x + 3|
\]

\[
|x - (-3)| < \sqrt[4]{1/1000}
\]

Let \( \delta = \sqrt[4]{1/1000} \)
6) Indicate how you can use the Intermediate Theorem to determine if the function

\[ f(x) = x^2 - x^4 + x \]

has a root over the interval \([1, 2]\)

In order to use the IVT we confirm that \(f(x)\) is a continuous function over the closed interval \([1, 2]\) since it is a polynomial. We now evaluate \(f(1) = 1\) and \(f(2) = -10\). By the IVT we can choose any \(y_0\) between \(f(1)\) and \(f(2)\) and there will exist a \(c\) in \([1, 2]\) so that \(f(c) = y_0\). We need to choose \(y_0 = 0\) which is possible since we have the interval \([-10, 1]\) on the \(y\)-axis. Therefore we have a \(c\) in \([1, 2]\) so that \(F(c) = 0\) which gives at least one root on the interval \([1, 2]\).