Math 1205 sol Homework 2 due on Friday

Use the formal definition of the limit with $\epsilon$ and $\delta$ to find the following limits.

You may work on this sheet or use your own paper.

Problem 1: **Verify the limit graphically**

If the limit is $\frac{1}{2} = .5$ then plot the points on either side of this limit on the $y$-axis which would be $y_1 = .5 + \epsilon$ and $y_2 = .5 - \epsilon$. Now solve for the $x$ values that would give you these $y$ values.

$.5 + \epsilon = \frac{1}{x}$ which gives $x = \frac{1}{.5 + \epsilon}$ and $.5 - \epsilon = \frac{1}{x}$ which gives $x = \frac{1}{.5 - \epsilon}$ around the point $x = 2$

Find $\delta_1$ and $\delta_2$: $\delta_1 = 2 - \frac{1}{.5 + \epsilon}$ and $\delta_2 = 1 - \frac{2\epsilon}{.5 - \epsilon}$, $\delta = \min(\delta_1, \delta_2) = \frac{2\epsilon}{.5 + \epsilon}$

Now plug in the epsilon you choose into this formula and see if the delta value is the same as the one you found.

Problem 2: **Verify the following limits algebraically and let $\epsilon = .01$**

a) \[
\lim_{x \to 5}(2x + 4) = 14
\]

If the limit exists then we have that

$|2x + 4 - 14| < .01$

$-.01 < 2x - 10 < .01$

$9.99 < 2x < 10.01$

$4.995 < x < 5.005$

$-.005 < x - 5 < .005$

$\delta_1 = -.005, \delta_2 = .005$, choose $\delta = .005$ so that

$|x - 5| < .005$

Therefore the definition of the limit is satisfied and limit does exist at $x = 5$.
b) \( \lim_{x \to 7} x^2 = 49 \)

\[ |x^2 - 49| < .01 \]

\[ -.01 < x^2 - 49 < .01 \]

\[ 48.99 < x^2 < 49.01 \]

\[ 6.9992 < x < 7.0007 \]

\[ -.000714 < x - 7 < .0007 \]

\( \delta_1 |-.000714|, \delta_2 = .0007 \), choose \( \delta = .0007 \) so that

\[ |x - 7| < .0007 \]

Therefore the definition of the limit is satisfied and limit does exist at \( x = 5 \).