1. Let \( \{e_1, e_2\} \) be the standard basis of \( \mathbb{C}^2 \), and let \( \alpha: \mathbb{C}^2 \to \mathbb{C}^2 \) be a \( \mathbb{C} \)-linear transformation satisfying \( \alpha(e_1 + ie_2) = 2e_1 \) and \( \alpha(e_1 - ie_2) = 2e_2 \). What is the matrix of \( \alpha \) with respect to the basis \( \{e_1, e_2\} \)? (2 points)

2. Let \((|)\) denote the standard scalar product on \( \mathbb{C}^d \) and let \( A \in M(d, \mathbb{C}) \) (the \( d \times d \) matrices, entries \( \mathbb{C} \)). Prove that \( (Au \mid Av) = (u \mid v) \) for all \( u, v \in \mathbb{C}^d \) if and only if \( AA^* = A^*A = I_d \) (the identity \( d \times d \) matrix). (3 points)

3. Let \( G = \langle g \rangle \) be the infinite cyclic group. Show that there exists a representation of \( G \) which is not unitarizable (consider one-dimensional representations). (2 points)

(3 problems, 7 points altogether)