Tenth Homework
Due 2:30 p.m., Monday May 2

1. Let $R$ be a commutative ring with a $1 \neq 0$, and let $S$ denote the set of nonzerodivisors of $R$ (that is $\{s \in R \mid sr \neq 0 \text{ for all } r \in R \setminus 0\}$).

(a) Prove that $S$ is a multiplicatively closed subset of $R$ which contains 1 but not 0.

(b) Prove that every element of $S^{-1}R$ is either a unit (that is, $u$ such that $uv = 1$ for some $v$) or a zero divisor (zero divisor includes 0).

2. Let $R$ be a commutative ring with a 1. Suppose there exist ring epimorphisms $\alpha : R \rightarrow \mathbb{Z}/2\mathbb{Z}$ and $\beta : R \rightarrow \mathbb{Q}$. Prove that there exists a ring epimorphism $\theta : R \rightarrow \mathbb{Z}/2\mathbb{Z} \times \mathbb{Q}$.

3. Let $R$ be an integral domain, let $S$ be a multiplicatively closed subset of $R$ which contains 1 but not 0, and let $p$ be a prime of $R$. Prove that $p/1$ is either a prime or a unit of $S^{-1}R$.

4. Exercise 9.4.7 on p. 311

(4 problems, 9 points altogether)