1205 ESP - Problem Set 25

1. Let \( f \) be continuous on \([a,b]\) and differentiable on \((a,b)\). Then
   
   i) If \( f'(x) > 0 \) for every \( x \) in \((a,b)\), then \( f \) is increasing on \((a,b)\).
   
   ii) If \( f'(x) < 0 \) for every \( x \) in \((a,b)\), then \( f \) is decreasing on \((a,b)\).

State the domains of each of the following functions and then find all critical points. Make a table to show your work and determine the intervals on which the function is increasing and the intervals on which the function is decreasing. Classify the local extrema.

a. \( g(t) = 4t^3 + 5t^2 - 42t + 7 \)

b. \( y = (2x - 1)^3 \)

c. \( f(x) = x \sqrt{x^2 - 9} \)

d. \( h(x) = x - \sqrt{x} \)

2. Find the absolute extrema of \( f \) on the given interval by finding all interior critical points and checking the functional values at those points and at the end points of the interval.

a) \( f(x) = \sin x + \cos^2 x ; \ [0,2\pi] \)

b) \( h(x) = x^2/4(x - 5) ; \ [-1,1] \)

c) \( g(x) = x \sqrt{2x - x^2} ; \ [0,2] \)

3. Find the maximum and minimum values of \( f(x) = x + \frac{4}{x} \) on \( 1 \leq x \leq 4 \).
4. Find \( \frac{dx}{dy} \) given: \( 3x^4 - 5y^2 = 8 - \sec(x^3) \)

5. Same directions as problem 1.
   
a) \( f(x) = \frac{x + 5}{2 - x^3 - 4} \)
   
b) \( f(x) = (2x - 4)^4(x - 1)^3 \)
   
c) \( f(x) = x\sqrt{2 - x} \)

6. Find \( \frac{dr}{d\theta} \) given: \( 5\theta + 3r^7 + \tan(r\theta) = r^2 \)

7. Let \( f(x) = 1 - x^2 \). Show that \( f(-1) = f(1) \) but there is no number \( c \) in \((-1,1)\) such that \( f'(c) = 0 \). Why does this not contradict the Rolle’s / Mean Value theorem?

8. The graph of the first derivative \( f' \) of a function \( f \) is shown.
   
a) On what intervals is \( f \) increasing? EXPLAIN.

   b) At what values of \( x \) does \( f \) have a local maximum or minimum? EXPLAIN.

   c) On what intervals is \( f \) concave upward or concave downward? EXPLAIN.

   d) What are the \( x \)-coordinates of the inflection points of \( f \)? WHY.