1205 ESP - Problem Set 7

1. Given \( f(x) = \frac{3x^2 + 14x + 8}{2x^2 + 6x - 8} \). Evaluate the following limits. Show all work.

a. \( \lim_{x \to 0} f(x) = \) ________________

b. \( \lim_{x \to 1} f(x) = \) ________________

c. \( \lim_{x \to -4} f(x) = \) ________________

2. Let \( f(x) = \begin{cases} 
  x^2 + 3, & x \geq 1 \\
  5x - 1, & x < 1 
\end{cases} \). Determine the following or explain why it does not exist.

a) \( \lim_{x \to 4} f(x) = \) ________________

b) \( \lim_{x \to 1^+} f(x) = \) ________________

c) \( \lim_{x \to 1^-} f(x) = \) ________________

d) \( \lim_{x \to 1} f(x) = \) ________________

3. Evaluate the following limits:

a. \( \lim_{x \to 3} 4 = \) _____

b. \( \lim_{x \to 3} x = \) _____.

c. \( \lim_{t \to 2} (t + 9) = \) __________

d. \( \lim_{w \to 2} (5w - 6)^2 = \) _____

e. \( \lim_{x \to 2} \frac{-5}{(x-2)^4} = \) __________

4. If \( f(x) = \sqrt{1-x}, \) \( x \leq 0, \) sketch the graph of \( f \) and state the domain and range. Then find the function \( f^{-1}(x), \) state its domain and range and sketch its graph.
5. a) If you have a graph of $y=f(x)$, explain how can you “see” right away that the function is one-to-one.

b) Explain what “one-to-one” means and why this “test” you use in part (a) works.

6. a. Describe in your own words what $|x-x_0|<\delta$ means.

b. Describe in your own words what $|f(x)-L|<\varepsilon$ means.

c. Show both of these inequalities on the appropriate axis (x or y-axis).

5. $f(x) = -\frac{1}{2}x^2 + 8$ Where $L=7$, $\varepsilon = .5$ and $x_0 = \sqrt{2}$.

a. Graph $f(x)$.

c. $|f(x)-L|<\varepsilon$. Show this restricted interval on the appropriate axis. Find the corresponding x-interval?

c. Now find $\delta$ so $|x-x_0|<\delta$ will imply $|f(x)-L|<\varepsilon$.

Could you have done this without using the graph? Explain why or why not.

6. Find the largest value of $\delta >0$ such that for all $x$ satisfying $0 < |x-x_0| < \delta$ $\implies$ $|f(x) - L| < \varepsilon$

$$\lim_{x \to 4} \sqrt{2x+1} = 3 \quad ; \quad \varepsilon = 0.1$$