Test 3 Review Problems

1) Graph the following functions using the scheme for graphing:
   a) \( f(x) = -x^3 + 4x^2 - 4x \)
   b) \( f(x) = \frac{x^2}{x^2 - 4} \)
   c) \( f(x) = \frac{x^3 + x - 9}{x} \)

2) Find the absolute extrema of the following functions on the given intervals:
   a) \( f(x) = -\sqrt{5 - x^2} \) on \([-\sqrt{5}, 0]\]
   b) \( f(x) = 4 - x^2 \) on \([-3, 1]\]

3) Find the Linearization of \( f(x) = \sqrt{x^2 + 16} \) at \( x = -3 \).

4) For \( y = 2x^3 - 3 \), find \( dy \) and \( \Delta y \) for \( x_0 = -1 \) and \( dx = .1 \).

5) A 216 m\(^2\) rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of its sides. What dimensions for the outer rectangle will require the smallest length of fence?

6) For each of the following graphs find the following:
   a) the intervals on which \( f(x) \) is increasing.
   b) the intervals on which \( f(x) \) is decreasing.
   c) the \( x \) values at which there is a local max.
   d) the \( x \) values at which there is a local min.
   e) the intervals on which \( f(x) \) is concave up.
   f) the intervals on which \( f(x) \) is concave down.
   g) the \( x \) values at which there is a point of inflection.

   ![Graph of \( f'(x) \)]
7) Does \( f(x) = x^2 - 4x + 1 \) satisfy the Rolle’s Theorem on the interval \([0,4]\). If it does, find the \( c \) that satisfies its conclusion. If it does not satisfy the hypotheses, which one does it violate?

8) Does \( f(x) = \frac{x}{x + 2} \) satisfy the Mean Value Theorem on the interval \([1,4]\). If it does, find the \( c \) that satisfies its conclusion. If it does not satisfy the hypotheses, which one does it violate?

9) Find the Taylor Polynomial of degree 3 centered at \( x = \pi \), for \( f(x) = \sin x \).