Test 2 Review

Sections 2.7 – 3.8; Worksheets 2 - 5

Derivatives:

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \] if the limit exists.

We evaluated this limit in several ways:

- Tables
- Reading from the graph
- Definition
- Rules

A function is differentiable if the derivative exists.

A derivative does not exist:
- At a corner
- At a discontinuity
- At a vertical tangent
- At a cusp

Know how to graph the derivative from the function and how to distinguish the function, derivative, and second derivative.

Know how to find higher order derivatives.

If \( f \) is differentiable at \( c \) then \( f \) is continuous at \( c \). But just because a function is continuous does not mean it is differentiable.

Right hand derivative at a point \( a \) is

\[ \lim_{h \to 0^+} \frac{f(a + h) - f(a)}{h}. \]

Left hand derivative at a point \( a \) is

\[ \lim_{h \to 0^-} \frac{f(a + h) - f(a)}{h}. \]

For a piecewise defined function check 1) to see if it is continuous. Then if it is continuous 2) test to see if the one sided derivatives are the same.

Know how to find the equations of tangent and normal lines.

Definition of \( e^x = e^{\ln a} \)

Since this limit is a rate of change it can give us instantaneous velocity:

If \( s = f(t) \) is the position function with respect to time, then

\[ v(t) = s'(t) = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t} \] is velocity

speed = \( |v(t)| \)

\( a(t) = v'(t) \) is acceleration

\( j(t) = a'(t) \) is jerk

We can use this information in many ways:

Make decisions like the following:

- Particle is momentarily at rest (changing directions) when \( v(t) = 0 \).
- Particle is moving to the right (forward) when \( v(t) \) is positive.
- Particle is moving to the left (backward) when \( v(t) \) is negative.
- Particle is speeding up when speed is increasing (velocity and acceleration have the same signs) and slowing down when speed is decreasing (velocity and acceleration have different signs).
- Particle is moving the fastest when speed is the greatest and moving the slowest when speed is zero.
The **linear density** \( \rho \) is the derivative of mass with respect to length, \( \rho = \frac{dm}{dx} \).

Know the following limits:

\[
\lim_{x \to 0} \frac{\sin x}{x} = 1
\]

\[
\lim_{x \to 0} \frac{\cos x - 1}{x} = 0
\]

Know all the differentiation rules on the derivative rules handout.

Know how to take a derivative using Implicit Differentiation.

To prove the rules for the inverse trig functions, and \( \log_a x \) use implicit differentiation.

To prove the rule for \( a^x \) use the chain rule.

To prove the rules for \( \tan x \), \( \cot x \), \( \sec x \), and \( \csc x \) use their relationships with \( \sin x \) and \( \cos x \).