I. Definitions and Notation

A. Second Derivative

1. **Def:** If \( f \) is a differentiable function, the second derivative is the derivative of the first derivative, i.e. \( \frac{d}{dx} \left[ \frac{dy}{dx} \right] \). The second derivative is the rate of change of the first derivative wrt \( x \) or you can interpret the second derivative as the rate of change of the rate of change.

2. Notation: \( f''(x), \ y'', \ \frac{d^2y}{dx^2}, \ D_x^2y, \ D_x^2f(x) \).

3. Acceleration: If \( s(t) \) is the position function of an object that moves in a straight line, its first derivative represents the velocity \( v(t) \) of the object as a fn of time, i.e.,
\[
v(t) = s'(t) = \frac{ds}{dt}.
\] The instantaneous rate of change of the velocity wrt time is called the acceleration \( a(t) \) of the object. Acceleration is the derivative of velocity and the second derivative of position, i.e.,
\[
a(t) = \frac{dv}{dt} = s''(t) = \frac{d^2s}{dt^2}.
\]

B. Third Derivative

1. **Def:** If \( f \) is a differentiable function, the third derivative is the derivative of the second derivative, i.e.
\[
\frac{d}{dx} \left[ \frac{d^2y}{dx^2} \right].
\] The third derivative is the rate of change of the second derivative wrt \( x \).

2. Notation: \( f'''(x), \ y''', \ \frac{d^3y}{dx^3}, \ D_x^3y, \ D_x^3f(x) \).

3. Jerk: If \( s(t) \) is the position function of an object that moves in a straight line, the third derivative of the position fn is the derivative of the acceleration fn and is called the jerk \( j \), i.e.
\[
j = a'(t) = \frac{da}{dt} = s'''(t) = \frac{d^3s}{dt^3}.
\]

C. The \( n^{th} \) Derivative

1. The process of differentiation can be continued. The \( n \)th derivative is obtained by differentiating \( f \) \( n \) times.

2. Notation of the \( n^{th} \) derivative: \( f^{(n)}(x), \ y^{(n)}, \ \frac{d^n y}{dx^n}, \ D^n f(x) \).
II. Examples:
   A. Find the indicated derivative
      1. \( y = 5x^2 + 19x - 2 \); \( \frac{d^3 y}{dx^3} \)

   *Note: The \((n+1)^{st}\) derivative of a polynomial of degree \(n\) is __________. 

      2. \( h(y) = 4y^5 - 4y^3 - 27 \); \( h^{(6)}(y) \)

      3. \( f(x) = e^{3x} \tan x \); \( f''(x) \)

      4. \( y = \sin(2x^3) \); \( \frac{d^2 y}{dx^2} \)

      5. Find the fourth derivative of \( g(t) = \frac{4}{t^4} \).
6. Find \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \) for \( \sqrt{x} + \sqrt{y} = 1 \)

B. Graphical Examples
In order to identify the function from the 1st derivative and the 2nd derivative, look for the following. Notice that \( f'(x) = 0 \) whenever \( y = f(x) \) has a horizontal tangent. Also \( f'(x) > 0 \) when \( y = f(x) \) has a positive slope and \( f'(x) < 0 \) when \( y = f(x) \) has a negative slope. We can also use this information to distinguish \( f' \) from \( f'' \), i.e., \( f''(x) = 0 \) whenever \( y = f'(x) \) has a horizontal tangent, etc.

Given the graphs below, determine \( f, f' \) and \( f'' \).
III. Physics

A. Rectilinear motion / Free Fall (Air resistance will be ignored)

1. Position Function: $s(t)$ – position as a function of time of a particle that is moving in a straight line.

2. Velocity function:
   a. Velocity is the rate of change of displacement $s$, with respect to time
   b. Formula: $v(t) = \frac{ds}{dt} = s'(t) = \lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t}$
   c. $v(t_0) > 0 \Rightarrow$ object is moving to the right / up.
   d. $v(t_0) < 0 \Rightarrow$ object is moving to the left / down.

2. Acceleration function: $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2} = s''(t) = \lim_{\Delta t \to 0} \frac{v(t + \Delta t) - v(t)}{\Delta t}$
   a. When $a(t) > 0 \Rightarrow$ velocity is increasing
      1.) $v(t) > 0 \Rightarrow$ object speeds up
         2.) $v(t) < 0 \Rightarrow$ object slows down
   b. When $a(t) < 0 \Rightarrow$ velocity is decreasing
      1.) $v(t) > 0 \Rightarrow$ object slows down
         2.) $v(t) < 0 \Rightarrow$ object speeds up
   c. Summary of the relationship between the signs of velocity and acceleration
      1.) When velocity and acceleration have the same algebraic sign, the object speeds up.
      2.) When velocity and acceleration have the different algebraic signs, the object slows down.

3. Jerk function: $j = a'(t) = \frac{da}{dt} = s''''(t) = \frac{d^3s}{dt^3}$

4. Displacement: $s = s(t + \Delta t) - s(t)$ (change in position – Not total distance)

5. Speed: $|v(t)| = \left| \frac{ds}{dt} \right| ; \text{ Speed} \geq 0 !!$
B. Example
1. Given the position function \( s(t) = \frac{1}{4} t^2 + \cos(3t) \) for \( 0 \leq t \leq 3 \), where \( s \) is measured in meters and \( t \) is measured in seconds.
   a. Determine the velocity, acceleration and jerk as a function of time.

b. Determine the position, velocity, acceleration and jerk at \( t = \frac{3\pi}{4} \)

c. Given the graph below, mark up the graph to indicate the following
   1.) Where is the body located on the axis at the beginning of the time interval? at the end?
   2.) When is the body momentarily at rest?
   3.) When does it move to the left? to the right?
   4.) When does it speed up? slow down?
   5.) When is it moving fastest? slowest?
   6.) When is it farthest from the axis origin?