Math 2214, Spring 2014, Form A

1. Which of the following is a linear equation?
   (a) \((t^2y'' + y')/y = t^3\).
   (b) \(y''' - y'' = ye^{y+t}\).
   (c) \(y' = t/y\).
   (d) \(y'' + y' = y^2\).

2. If \(x' = x^2/t\), and \(x(1) = 1\), then \(x(2)\) is
   (a) \(1/(1 - \ln 2)\).
   (b) \(1/(1 + \ln 2)\).
   (c) \(\sqrt{6}\).
   (d) \(\sqrt{2}\).

3. A lake contains \(10^8\) gallons of water. The stream passing through the lake transports a constant \(10^6\) gallons per day. A plant situated just upstream of the lake starts releasing 100 grams per day of a toxic chemical. Let \(Q\) denote the amount of chemical in the lake measured in grams, and \(t\) the time in days measured from the time when the pollution started. Assume that the lake is well mixed. Then \(Q\) satisfies the following equation
   (a) \(Q' = 10^6 - Q/10^8, Q(0) = 100\).
   (b) \(Q' = 100 - Q/100, Q(0) = 10^8\).
   (c) \(Q' = 100 - Q/100, Q(0) = 0\).
   (d) \(Q' = -Q/100, Q(0) = 100\).
4. For the system

\[ \begin{align*}
  x' &= -x + 5y, \\
  y' &= -7x + y,
\end{align*} \]

the origin is a

(a) focus.
(b) saddle.
(c) center.
(d) node.

5. The general solution of the system \( y' = Ay \), where

\[ A = \begin{pmatrix} 0 & 4 \\ 1 & 0 \end{pmatrix}, \]

is

(a) \( c_1 e^{-2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \).
(b) \( c_1 e^{-2t} \begin{pmatrix} -2 \\ 1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \).
(c) \( c_1 \cos(2t) \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 \sin(2t) \begin{pmatrix} 1 \\ -2 \end{pmatrix} \).
(d) \( c_1 t e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \).

6. A particular solution of the equation \( y'' + y = e^t/t \) should have the form

(a) \( Ae^t/t \).
(b) \( Ae^t/t^2 \).
(c) \( Ae^t + Be^t/t \).
(d) \( u(t) \sin t + v(t) \cos t \).
7. Which of the following is a direction field for the equation $y' = y + t^2$?

(a) 

(b) 

(c)
8. Which of the following is not a particular solution of the equation $y^{''''} - y = e^t$?

(a) $te^t/4 + 5e^t$.
(b) $te^t/4 + e^{-t}$.
(c) $te^t/4 + t^2e^t + 7e^t$.
(d) $te^t/4 + 6e^t + 2\sin t$.

9. You solve the initial value problem $y' = y^2 + t$, $y(0) = 1$ using the Euler method with $h = 0.2$. Then the approximation you find for $y(0.4)$ is

(a) 1.24.
(b) 1.6275.
(c) 1.528.
(d) 1.2.

10. The matrix

$$A = \begin{pmatrix}
1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

has how many linearly independent eigenvectors for the eigenvalue 1?

(a) One.
(b) Three.
(c) Four.
(d) Two.
11. Which of the following plots shows the solution of the problem $y'' + 25y = \cos(5t)$, $y(0) = 1$, $y'(0) = 0$?

(a) 

(b) 

(c)
12. A nonlinear system is given by

\[ x'_1 = x_1 x_2 - x_1, \]
\[ x'_2 = x_2 x_1 - x_1^2. \]

The linearization at the equilibrium point \((1, 1)\) is the system

(a)  
\[ y'_1 = y_2, \]
\[ y'_2 = y_2. \]

(b)  
\[ y'_1 = -y_1 + y_2, \]
\[ y'_2 = -y_1 + y_2. \]

(c)  
\[ y'_1 = y_1 + y_2, \]
\[ y'_2 = y_2. \]

(d)  
\[ y'_1 = y_1 + y_2, \]
\[ y'_2 = -y_1 + y_2. \]