Math 2015 Test 1      Spring, 2007

Instructions:
Answer the 10 multiple choice questions using a #2 pencil.
Answer the free response in the space provided. You must justify all work and only use methods discussed in class.
Write your name and your student ID number at the top of the page, and sign the pledge. I will neither give nor receive unauthorized assistance on this exam.

Signature:  _____________________________________________

Free Response:

1. State the fundamental theorem of calculus version I and II.

   Version I. (3 pts):  

   Version II. (3 pts):

2. The rate of pollution pouring into a lake is measured every 5 days, with results in the following table.

<table>
<thead>
<tr>
<th>Time in days</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Pollution (tons/day)</td>
<td>4</td>
<td>6</td>
<td>13</td>
<td>11</td>
<td>10</td>
</tr>
</tbody>
</table>

   Estimate the amount of pollution that has entered the lake in the twenty days according to the following:

   a. (3 pts) Using Left-Hand Sums:

   b. (3 pts) Using Right-Hand Sums:

   c. (2 pts) Give a better estimate:

3. Choose the smallest \( n \) possible and state whether your answer will be an overestimate, exact or an underestimate.

   a- (2 pts) If we have the integral of a linear function to approximate using the trapezoidal Rule, we should let \( n = ____ \) and our answer will be ________.

   b- (2 pts) If we have the integral of a quadratic function to approximate using Simpson’s Rule, we should let \( n = ____ \) and our answer will be ________.
4. Approximate the integral \( \int_1^3 (t^2 + t) \, dt \) using
   
   a. SIMPSON approximation with \( n = 4 \),
      
      i- (1 pts) Find the \( \Delta t \)
      
      ii- (2 pts) Set up the correct formula
      
      iii- (1 pts) Calculate the approximation
   
   b. The Trapezoid approximation with \( n = 4 \),
      
      i- (1 pts) Find the \( \Delta t \)
      
      ii- (2 pts) Set up the correct formula
      
      iii- (1 pts) Calculate the approximation

5. (5 pts) Which of the following is a solution to the differential equation \( y' - y = 0 \)? (Show your work)
   
   1. \( y = e^{-x} \)  
   2. \( y = xe^x \)  
   3. \( y = e^{2x} \)  
   4. \( y = 2e^x \)

6. Given the following graph of \( f(x) \) and the areas of the shaded regions, do (a) and (b).

   a. (3 pts) Evaluate \( \int_{-3}^{12} f(x) \, dx \)
   
   b. (3 pts) Evaluate \( \int_{-6}^{6} f(x) \, dx \)
7. (4 pts) The graph of \( f(x) = x^2 + x \) and \( g(x) = x + 1 \) is shown below. **Set up** the definite integral to find the area between the curves.

![Graph of f(x) and g(x)](image)

8. (3 pts) If a dripping faucet leaks water at the rate of \( r(t) \) ounces / minute, where \( t \) is measured in minutes, write a definite integral that expresses the total amount of water that leaks out of the faucet over an 8-hour night of sleep. (Include units)

9. The graphs of \( f(x) \) and \( g(x) \) are shown below.

   ![Graphs of f(x) and g(x)](image)

   a- (3 pts) What is \( \int_{0}^{3} f(x) \, dx \) ?

   b- (3 pts) What is \( \int_{0}^{3} g(x) \, dx \) ?

   (Hint: What does the picture of \( g(x) \) look like?)
MULTIPLE CHOICE:

The 10 multiple choice questions are 5 pts each.

1. Let \( F(x) = \int_0^x te^{-t} \, dt \). Then \( F'(x) = \)

   (1) \( te^{-t} \)  (2) \( te^{-x} \)  (3) \( xe^{-x} \)  (4) \( \int_0^x xe^{-x} \, dx \)

2. Suppose we have found that the general solution to a differential equation is \( y(t) = Ce^{2t} \). If \( y(2) = 3 \), then \( C = ? \)

   (1) 3  (2) 2  (3) \( \frac{\ln 2}{3} \)  (4) \( 3e^{-4} \)

3. Suppose \( f(-2) = 8 \) and \( f(1) = -3 \). Then \( \int_{-2}^{1} f'(x) \, dx \) is?

   (1) -11  (2) 11  (3) -3  (4) 8

Problems 4, 5, 6 and 7 refer to the graph below which shows the rate \( r(t) \) at which water flows into or out of a reservoir. Positive rates correspond to water flowing into the reservoir. The units for \( r(t) \) are hundreds of gallons per day.

4. At \( t = 4 \) days, is the amount of water in the reservoir increasing, decreasing, or not changing?

   (1) Increasing  (2) Decreasing  (3) Not changing  (4) Not enough information to know.

5. Over the period from \( t = 0 \) to \( t = 6.3 \), at which time \( t \) does the reservoir contain the most water?

   (1) \( t = 0 \)  (2) \( t = 1.3 \)  (3) \( t = 2.5 \)  (4) \( t = 6.3 \)

6. At \( t = 0 \) there are 5 hundred gallons in the reservoir. If we determine that \( \int_0^1 r(t) \, dt \approx -2 \), how many gallons of water are in the reservoir at \( t = 1 \)?

   (1) 300  (2) 498  (3) 502  (4) 500
7. Is there MORE or LESS water at time \( t = 6.3 \) than at time \( t = 0 \), and how can we tell? You must decide both “more” or “less” and the correct reason.

(1) LESS, because \( r(6.3) = 0 \), so there is no water in the reservoir by \( t = 4 \).

(2) LESS, because there is more area beneath the axis than above, indicating more water has left than entered from \( t = 0 \) to \( t = 6.3 \).

(3) MORE, because there is more area above the axis than beneath, indicating more water has entered than left from \( t = 0 \) to \( t = 6.3 \).

(4) MORE, because \( r(t) \) has been increasing since \( t = 1 \), and in fact increases more often on \( t = 0 \) to \( t = 6.3 \) than it decreases.

8. Let \( Q(t) \) be the quantity of fish in a lake after \( t \) months. Suppose 7\% of the fish in the lake are caught each month, and 120 new fish enter the lake each month. Which of the following differential equations would describe the function \( Q(t) \)?

\[ Q' = 120t - 0.07t \quad \text{(1)} \]
\[ Q' = 120Q + 0.07Q \quad \text{(2)} \]
\[ Q' = 120 - 0.07Q \quad \text{(3)} \]
\[ Q' = 120t - 0.07Q \quad \text{(4)} \]

9. Given: \( y' = 2y - t \) and \( y(2) = -1 \). Use Euler’s method to estimate \( y(3) \). \( y(3) = \)

(1) -4 \quad (2) -5 \quad (3) 0 \quad (4) -2 - t

10. Suppose we set up a right-hand sum with \( n \) subintervals to approximate the integral \( \int_{0}^{3} \sqrt{9 - x^2} \, dx \). The function \( y = \sqrt{9 - x^2} \) is graphed below on \([0, 3]\).

What could we say about the relationship between the right-hand sum and the value of the integral?

1. The right-hand sum is less than the value of the integral.
2. The right-hand sum is greater than the value of the integral.
3. The right-hand sum is the exact value of the integral.
4. There is nothing here to suggest whether the right-hand sum would be greater than, less than or equal to the integral.

**Bonus.**

(2 pts) What is the relation between the left hand sum, the right hand sum and the Trapezoid rule?

(2 pts) Is \( \int_{a}^{b} f(x) \, dx \) always equal to the area between \( f(x) \) and the \( x \)-axis on \([a, b]\)? Explain your answer.