Math 2015  Review Problems for Test 1

Note: This is NOT a complete listing of all types of problems that will be on the test. You will need to review all homework problems and all class notes to be fully prepared. Lab 1 is also covered on the test.

1) If a dripping faucet leaks water at the rate of \( r(t) \) ounces/minute, where \( t \) is measured in minutes, write a definite integral that expresses the total amount of water that leaks out of the faucet over an 8-hour night of sleep.

2) Neatly sketch a function \( f(x) \) such that \( \int_{a}^{b} f(x) \, dx = 0 \).

3) Multiple Choice. In general, using Left-hand sums which of the following will give us the best approximations for the value of a definite integral \( \int_{a}^{b} f(t) \, dt \)?
   (a) Large \( \Delta t \), large \( n \)
   (b) Large \( \Delta t \), small \( n \)
   (c) Small \( \Delta t \), large \( n \)
   (d) Small \( \Delta t \), small \( n \)

4) Suppose \( f(t) \) and \( g(t) \), graphed below, represent the number of trees chopped down per year in two neighboring forest areas over a 12-year period.

   ![Forest Depletion Graph]

   a) Approximate \( \int_{0}^{12} f(t) \, dt \) using \( \Delta t = 2 \). You may use any method you wish here. Include units.
   b) In a complete sentence, explain what the above value represents in the context of the forest.
   c) Approximate the year in which the total number of trees chopped down during this 12-year period of time is the same for both forests.
   d) In a complete sentence, explain how you came to your conclusion for part (c).
   e) If both forests started with the same number of trees, which forest would have the most trees left at time \( t = 4 \) years? In a complete sentence, explain your conclusion.

5) True or False. Using the right hand sum to approximate \( \int_{a}^{b} f(x) \, dx \) will always give you an overestimate.
6) Consider a sports car which accelerates from 0 ft/sec to 88 ft/sec in 5 seconds (88 ft/sec = 60 mph). The car's velocity is given in the table below.

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>V(t)</td>
<td>0</td>
<td>30</td>
<td>52</td>
<td>68</td>
<td>80</td>
<td>88</td>
</tr>
</tbody>
</table>

a) Sketch a graph of the velocity of the car over these 5 seconds. Label both axes and the units.
b) Use Δt = 1. Draw rectangles on the graph so that the total area of the rectangles would give a lower estimate for the distance the car travels in 5 seconds. Note whether you are using left or right-hand rectangles here.
c) What is the minimum distance this car traveled in 5 seconds.
d) Use the trapezoidal rule to find a better estimate for the total distance the car travels in 5 seconds.

7) Suppose the predicted rate of growth, in dollars per year, of the net worth, \( f(t) \), of a company is given by \( f'(t) = 4800 - 12t^2 \), where \( t \) is measured in years since 1990. Suppose also that the net worth of the company at any time after 1990 is given by \( f(t) = 4800t - 4t^3 + 40000 \). The rate of growth, \( f'(t) \), is plotted below.

![Growth Rate of a Company](image)

a) Is the net worth of the company increasing or decreasing between 1990 and 2000?
b) Would \( \int_{20}^{30} f'(t) \, dt \) be positive or negative? Explain (in a complete sentence) what this means in terms of the company's net worth. Be certain that you correctly interpret the numbers 20 and 30 and the value of the net worth. Be sure to use appropriate units, when needed.

8) If we want to approximate \( \int_{5}^{9} f(t) \, dt \) using left-hand sums and \( n=8 \) subintervals, how wide, \( \Delta t \), must each subinterval be?

9) The fundamental theorem of calculus is

a) \( \int_{a}^{b} f'(x) \, dx = f(b) - f(a) \)  
   c) \( \int_{a}^{b} f'(x) \, dx = f'(a) - f'(b) \)

b) \( \int_{a}^{b} f(x) \, dx = f(b) - f(a) \)  
   d) \( \int_{a}^{b} f(x) \, dx = f(b) - f(a) \)

10) If \( \int_{0}^{5} g'(x) \, dx = 15 \) and \( g(0) = 4 \), find \( g(5) \).
11) If a marathon runner wears out his shoes at a rate given by $s'(t)$ pairs of shoes per year, where $t$ is measured in years, write a complete sentence explaining what $\int_0^5 s'(t)\,dt$ would represent. Include units.

12) After a foreign substance is introduced into the blood, the rate at which antibodies are made is given by

$$r(t) = \frac{t}{t^2 + 1}$$

thousands of antibodies per minute,

where time, $t$, is measured in minutes and $0 \leq t \leq 4$.

a) What is the rate at which antibodies are made 2 minutes after the substance is introduced into the blood? Include units.

b) Assuming there are no antibodies present at time $t = 0$, estimate the total quantity of antibodies in the blood at the end of 4 minutes. Use any technique you learned in this class. Tell me which technique you chose. Include units with your answer.

13) To estimate the land area of her backyard, a homeowner takes measurements every 50 meters across the width of the land. The following data is collected

<table>
<thead>
<tr>
<th>Distance (meters) (on horiz. axis)</th>
<th>Distance (meters) (on vertical axis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>113</td>
</tr>
<tr>
<td>100</td>
<td>80</td>
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<td>150</td>
<td>155</td>
</tr>
<tr>
<td>200</td>
<td>163</td>
</tr>
<tr>
<td>250</td>
<td>120</td>
</tr>
<tr>
<td>300</td>
<td>0</td>
</tr>
</tbody>
</table>

A sketch of the land follows:

Estimate the total amount of land the woman owns using Simpson’s Rule. Include units.
14) Given the following graph of f(x) and the areas of the shaded regions, do (a) and (b).

\[
\begin{array}{cccccc}
A = 25 & A = 12 & A = 11 & A = 14
\end{array}
\]

a) Evaluate \( \int_{6}^{9} f(x) \, dx \)
b) Evaluate \( \int_{3}^{9} f(x) \, dx \)

15) Snow is forming on the ground at a rate given by \( \frac{dy}{dt} = 2\sqrt{t} + 1 \) inches per hour where \( y \) is the depth of the snow in inches at time \( t \) measured in hours since the snow started forming.

a) Use four left-hand rectangles to make an under-approximation of how much snow fell between 2 hours and 4 hours after the snow began to fall.
b) Represent your answer to part (a) on a graph. A sketch of \( \frac{dy}{dt} \) is below:

c) Suppose now that you are also given that \( y(t) = \frac{4}{3}t^{3/2} + t \). Use \( y \) to determine exactly how much snow fell between 2 hours and 4 hours after the snow began to fall.

16) Suppose we define \( F(x) = \int_{1.7}^{x} \cos(\sqrt{t}+1) \, dt \). Find \( F'(x) \).
17) Find the particular solution to \( y' = -3y \), and \( y(1) = 5 \).

18) Suppose \( y'(t) = (y(t))^2 \) and \( y(0) = 2 \). Using Euler’s Method, estimate \( y(1) \) and \( y(2) \).

19) Is \( y = \sqrt{x} \) a solution to \( y' = \frac{1}{2}y \)? Is it a solution to \( 2y'(t) = \frac{1}{y(t)} \)? Why or why not?