Practice for Test I

1. (#2, p. 201, Hughes-Hallet custom first edition revised) Annual coal production in the US (in quadrillion BTU per year) is given in the following table. Estimate the total amount of coal produced in the US between 1960 and 1990. (Including correct units.)

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</thead>
<tbody>
<tr>
<td>Rate of production</td>
<td>10.82</td>
<td>13.06</td>
<td>14.61</td>
<td>14.99</td>
<td>18.60</td>
<td>19.33</td>
<td>22.46</td>
</tr>
</tbody>
</table>

If the rate of change is given by a function $C(t)$, where $t$ is in years and $C$ is in quadrillion BTU per year, write down the integral that represents the total coal production from 1960 to 1990. Explain why your integral represents this quantity.

2. (#11, p. 201, Hughes-Hallet custom first edition revised) The rate of growth of the height of two species of trees is shown in the figure below, where $t$ is measured in years, and the rate is given in feet per year. If the two trees are the same height at time $t = 0$, which tree is taller after 5 years? After 10 years? Why?

3. If $F(x) = \int_{-3}^{x} \sin(1 - t^2) \, dt$, what is $F'(x)$?

4. Estimate the integral $\int_{-1}^{3} x^2 - 1 \, dx$
   
   (a) Using a left hand sum and $n = 8$ subintervals.
   (b) Using the trapezoidal rule and $n = 3$ subintervals.
   (c) Using Simpson’s rule and $n = 4$ subintervals.

   It can be shown that if $F(x) = \frac{1}{3}x^3 - x$, $F'(x) = x^2 - 1$. Use this fact and the Fundamental Theorem (Version I) to find the exact value of $\int_{-1}^{3} x^2 - 1 \, dx$.

5. For the following function as marked on the axes given, estimate graphically the value of the integral from $x = 0$ to $x = 12$. 
6. Suppose the rate of change of a casserole’s temperature in an oven $t$ minutes after it was put in is given by

$$f'(t) = 2(.99)^t.$$ 

If the temperature when the casserole was put into the oven was $75^\circ$, write down an expression which (when evaluated) will give the temperature 15 minutes later.

7. Show that $y = 3e^{-t} + 2$ is a solution to $y' = 2 - y$.

8. Given the initial value problem $y' = y^2 - t$, $y(0) = -1$, estimate the values of $y(1)$ and $y(2)$ using Euler’s method. (Use a step size of $h = 1$.)