**Form A**

**Instructions:** Fill in A, B or C in the Test Version section. Then enter your NAME, ID Number, CRN (under Class ID) and write A, B, or C (under Test ID) on the opscan sheet. Darken the appropriate circles below your ID number and Class ID (CRN). **Use a number 2 pencil.** Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1 – 14 of the op scan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the op scan sheet with your answers, this exam and all scrap paper at the end of this part of the final exam.

**Exam Policies:** You may not use a book, notes, formula sheet, or a calculator or computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: ________________________________

Name (printed): ________________________________

Student ID #: ________________________________
1. Evaluate \( \int_1^e \ln \sqrt{x} \, dx \)

(A) \( \frac{1}{8} \) \hspace{1cm} (B) \( \frac{1}{4} \) \hspace{1cm} (C) \( \frac{1}{2} \) \hspace{1cm} (D) 1

2. The area of the region bounded by \( x = \sqrt{3y}, x + y = 6 \) and the \( y \)-axis is

(A) \( \frac{18\sqrt{8}}{3} \) \hspace{1cm} (B) \( \frac{3}{2} \) \hspace{1cm} (C) \( \frac{36\sqrt{8}}{3} \) \hspace{1cm} (D) \( \frac{21}{2} \)

3. If the work required to stretch a spring 1 foot beyond its natural length is 9 foot–lbs, how much work is needed to stretch it 3 inches beyond its natural length?

(A) 18 foot–lbs \hspace{1cm} (B) \( \frac{9}{4} \) foot–lbs \hspace{1cm} (C) \( \frac{9}{16} \) foot–lbs \hspace{1cm} (D) 36 foot–lbs

4. Evaluate \( \int \frac{x + 11}{(x + 1)(x^2 + 4)} \, dx \)

(A) \( 2 \ln |x + 1| - \ln(x^2 + 4) + 3 \arctan(x) + C \)

(B) \( 2 \ln |x + 1| - \ln(x^2 + 4) + \frac{3}{2} \arctan \left( \frac{x}{2} \right) + C \)

(C) \( 2 \ln |x + 1| - \ln(x^2 + 4) + 3 \arctan(x^2 + 4) + C \)

(D) \( 2 \ln |x + 1| + 2 \ln(x^2 + 4) + C \)
5. You are allowed to use only ONE integral to determine the volume of the solid of revolution given by the following region rotated about \( y = -1 \). Which method will work?

(A) The Shell Method only; the Washer Method requires more than one integral.
(B) The Washer Method only; the Shell Method requires more than one integral.
(C) Both the Shell Method and the Washer Method require only one integral.
(D) It is impossible to determine the volume of this solid of revolution with only one integral.

6. Consider the function

\[
f(x) = \begin{cases} 
0, & x \leq 2 \\
ke^{-x/2}, & x > 2 
\end{cases}
\]

For what value of \( k \) will \( f(x) \) be a probability density function?

(A) \( \frac{1}{2} \)  (B) \( \frac{e}{2} \)  (C) \( 2e \)  (D) No such \( k \) exists.

7. Let \( a \) be a real number. Suppose that the tangent line to the curve \( x(t) = t^4, \ y(t) = at^2 \) at the point when \( t = 1 \) also passes through the point \( (2, 6) \). What is the value of \( a \)?

(A) 1  (B) 2  (C) 4  (D) 10
8. Find the x-coordinate (i.e., $\bar{x}$) of the centroid (center of mass with constant density) of the region shown below. Use the fact that the area of the region is equal to $\frac{1}{3}$.

\[ y - 4x^2 = 0 \]
\[ y - x^2 = 0 \]

(A) $\frac{1}{5}$  
(B) $\frac{3}{8}$  
(C) $\frac{9}{16}$  
(D) $\frac{3}{4}$

9. A student claims that both of the following parametric equations have the same graph as $y = x^2$.

$I$ : $x_1 = t^2$  
$y_1 = t^4$

$II$ : $x_2 = t^3$  
$y_2 = t^6$

Which of the following is true?

(A) The student is correct. Both $I$ and $II$ have the same graph as $y = x^2$.
(B) The student is incorrect. Only $I$ has the same graph as $y = x^2$. $II$ has a different graph.
(C) The student is incorrect. Only $II$ has the same graph as $y = x^2$. $I$ has a different graph.
(D) The student is incorrect. Neither $I$ nor $II$ has the same graph as $y = x^2$.

10. The series \[ \sum_{n=1}^{\infty} \frac{n + 3}{5n + n^2} \]

(A) diverges by the Ratio Test.
(B) diverges by the Limit Comparison Test with $b_n = 1/n$.
(C) converges by the Limit Comparison Test with $b_n = 1/n$.
(D) converges because $\lim_{n \to \infty} \frac{n + 3}{5n + n^2} = 0$. 
11. To approximate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n10^n}$ with an error smaller than $10^{-3}$, one needs

(A) to add at least the first two terms of the series.
(B) to add at least the first three terms of the series.
(C) to add at least the first four terms of the series.
(D) to add at least the first five terms of the series.

12. Which of the following series converge absolutely?

$I : \sum_{n=1}^{\infty} \frac{(-3)^n (n+1)!}{(2n+1)!}$

II : $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(A) only I
(B) only II
(C) both I and II
(D) neither I nor II

13. If $\sum_{k=1}^{n} a_k = \frac{n+1}{n}$, then $\lim_{n \to \infty} a_n =$

(A) 0
(B) 1
(C) $\infty$
(D) The limit cannot be determined.

14. The integral $\int_{0}^{1/2} \frac{1}{1-x^4} \, dx$ can be written as which of the following power series?

(A) $\sum_{n=0}^{\infty} \frac{1}{4n} \left( \frac{1}{16} \right)^n$
(B) $\sum_{n=0}^{\infty} \frac{1}{8n} \left( \frac{1}{16} \right)^n$
(C) $\sum_{n=0}^{\infty} \frac{1}{4n+1} \left( \frac{1}{16} \right)^n$
(D) $\sum_{n=0}^{\infty} \frac{1}{8n+2} \left( \frac{1}{16} \right)^n$