Instructions: Please enter your NAME, ID Number, FORM DESIGNATION, and your CRN on the opscan sheet. The CRN should be written in the box labeled COURSE. Do not include the course number. Darken the appropriate circles below your ID number and below the Form designation letter. Use a number 2 pencil. Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1 - 15 of the op scan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the op scan sheet with your answers and the question sheets, including this cover sheet, at the end of this part of the final exam.

Exam Policies: You may not use a book, notes, formula sheet, or a calculator or computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: __________________________________________

Name (printed): ______________________________________

Student ID #: ________________________________________

1. Which one of the listed table integral formulas

A: \( \int \frac{u^2}{\sqrt{a^2 + u^2}} \, du = -\frac{a^2}{2} \ln \left( u + \sqrt{a^2 + u^2} \right) + \frac{u\sqrt{a^2 + u^2}}{2} + C \)

B: \( \int \frac{1}{u^2 \sqrt{a^2 + u^2}} \, du = -\frac{\sqrt{a^2 + u^2}}{a^2u} + C \)

C: \( \int \frac{1}{u \sqrt{a^2 + u^2}} \, du = -\frac{1}{a} \ln \left( \frac{a + \sqrt{a^2 + u^2}}{u} \right) + C \)

can be applied to the integral below after an appropriate substitution?

\( \int \frac{1}{\sqrt{1 + \left( \frac{1}{x} \right)^2}} \, dx \)

(1) A  (2) B  (3) C  (4) None of them

2. Evaluate \( \lim_{t \to 0} \frac{\sin(t^2)}{1 - \cos(t)} \).

(1) Does not exist  (2) 1  (3) \( \frac{-1}{2} \)  (4) 2
3. An electronic device in a gasoline pump measures the pumping rate \( V'(t) \) in gallons per second. The table below lists the output of this device when you were fueling your car. Use Simpson’s rule to estimate the amount of gasoline \( V \) that you have pumped in your car.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V'(t) )</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.15</td>
<td>0.15</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(1) \( \frac{1}{3}(0.2 + 4 \cdot 0.3 + 2 \cdot 0.2 + 4 \cdot 0.15 + 2 \cdot 0.15 + 4 \cdot 0.1 + 0.1) \)

(2) \( 1 \cdot (\frac{1}{2} \cdot 0.2 + 0.3 + 0.2 + 0.15 + 0.1 + \frac{1}{2} \cdot 0.1) \)

(3) \( \frac{1}{3}(0 \cdot 0.2 + 1 \cdot 0.3 + 2 \cdot 0.2 + 3 \cdot 0.15 + 4 \cdot 0.15 + 5 \cdot 0.1 + 6 \cdot 0.1) \)

(4) \( \frac{1}{2}(0.2 + 2 \cdot 0.3 + 2 \cdot 0.2 + 2 \cdot 0.15 + 2 \cdot 0.15 + 2 \cdot 0.1 + 0.1) \)

4. Evaluate the integral \( \int_0^\infty xe^{-x^2} \, dx \).

(1) \( \frac{1}{e} \)   \hspace{1cm} (2) \( 1 \)   \hspace{1cm} (3) \( \frac{1}{2} \)   \hspace{1cm} (4) Integral diverges

5. Find the curve \( y = f(x) \) that passes through the point \((4,9)\) and whose slope at each point is \( 3\sqrt{x} \).

(1) \( f(x) = 2x^{3/2} - 7 \)   \hspace{1cm} (2) \( f(x) = 2x^{3/2} - 9 \)

(3) \( f(x) = 3x^{3/2} - 15 \)   \hspace{1cm} (4) \( f(x) = 3\sqrt{x} - 6 \)

6. Suppose \( f(x) = e^x + c \) where \( c \) is a constant. Suppose also that the average value of \( f \) over \([0, \ln 2]\) is \( \frac{1}{\ln 2} \). Then,

(1) \( c = -\frac{1}{\ln 2} \)   \hspace{1cm} (2) \( c = -\frac{1}{(\ln 2)^2} - \frac{1}{\ln 2} \)

(3) \( c = \frac{1}{(\ln 2)^2} - \frac{2}{\ln 2} \)   \hspace{1cm} (4) \( c = 0 \)

7. Evaluate \( \int_1^3 \frac{(x+1)(x-1)}{x^2} \, dx \).

(1) \( \frac{2}{3} \)   \hspace{1cm} (2) \( \frac{4}{3} \)   \hspace{1cm} (3) \( 2 \)   \hspace{1cm} (4) \( \frac{8}{3} \)
8. Find $\frac{dy}{dx}$ if $y = \int_{\cos(x)}^{1} \frac{1}{\ln(t)} \, dt$.

\begin{align*}
(1) \quad & \ln\left( \ln(\cos(x)) \right) \\
(2) \quad & -\frac{\sin(x)}{\ln(\cos(x))} \\
(3) \quad & \frac{\sin(x)}{\ln(\cos(x))} \\
(4) \quad & -\frac{1}{\ln(\cos(x))}
\end{align*}

9. Suppose the curves $f(x)$ and $g(x)$ only intersect at $x = -2, x = 0,$ and $x = 2$. It is known that

$$\int_{-2}^{0} (f(x) - g(x)) \, dx = -3 \quad \text{and} \quad \int_{0}^{2} (f(x) - g(x)) \, dx = 5.$$ 

What is the total area of the regions bounded by $f(x)$ and $g(x)$?

\begin{align*}
(1) \quad & 2 \\
(2) \quad & \frac{5 - 3}{2} \\
(3) \quad & 8 \\
(4) \quad & \frac{5 + 3}{2}
\end{align*}

10. A tank with the shape of an inverted right circular cone (the vertex is down) is filled with milk (weighing $64.5 \text{ lb/ft}^3$) to a depth of one half its height. If the height is 20 ft. and its diameter is 8 ft., find the work done by pumping all of the milk to the top of the tank. Assume the origin is at the vertex.

\begin{align*}
(1) \quad & 64.5\pi \int_{0}^{10} (5y)^2 (20 - y) \, dy \\
(2) \quad & 64.5\pi \int_{0}^{10} \left( \frac{y}{5} \right)^2 (20 - y) \, dy \\
(3) \quad & 64.5\pi \int_{10}^{20} \left( \frac{y}{5} \right)^2 (20 - y) \, dy \\
(4) \quad & 64.5\pi \int_{10}^{20} \left( \frac{y}{5} \right)^2 (y - 10) \, dy
\end{align*}

11. Use the shell method to find the volume of the solid generated by a region bounded by the graphs $y = \sqrt{x}, x = 4$ and $y = 0$ revolved around $y = 2$.

\begin{align*}
(1) \quad & 2\pi \int_{0}^{4} (y)(4 - y)^2 \, dy \\
(2) \quad & 2\pi \int_{0}^{2} (2 - y)(4 - y^2) \, dy \\
(3) \quad & 2\pi \int_{0}^{4} \left( 4 - (2 - \sqrt{x})^2 \right) \, dx \\
(4) \quad & \pi \int_{0}^{4} (2 - \sqrt{x})^2 \, dx
\end{align*}
12. Calculate the arc length of the graph \( y = \sin(x) \) on the interval \([0, \pi]\).

\[
\begin{align*}
(1) & \quad \int_0^\pi (1 + \cos^2(x))^2 \, dx \\
(2) & \quad \int_0^{\pi/2} \sqrt{1 + \sin^2(x)} \, dx \\
(3) & \quad \int_0^\pi \sqrt{1 + \cos^2(x)} \, dx \\
(4) & \quad \int_0^\pi \sqrt{1 + \frac{\sin^2(x)}{x^2}} \, dx
\end{align*}
\]

13. Let region \( R \) be bounded by \( x = 2y + 5 \) and \( x = y^2 + 2 \). The moment with respect to the \( y \)-axis of region \( R \) with constant density \( \delta = 2 \) is:

\[
\begin{align*}
(1) & \quad \frac{\int_{-1}^3 y(3 + 2y - y^2) \, dy}{\int_{-1}^3 3 + 2y - y^2 \, dy} \\
(2) & \quad 2 \int_{-1}^3 y(3 + 2y - y^2) \, dy \\
(3) & \quad \frac{\int_{-1}^3 ((2y + 5)^2 - (y^2 + 2)^2) \, dy}{2 \int_{-1}^3 3 + 2y - y^2 \, dy} \\
(4) & \quad \int_{-1}^3 ((2y + 5)^2 - (y^2 + 2)^2) \, dy
\end{align*}
\]

14. Evaluate \( \int \frac{25x^3 + 16x - 8}{25x^2 + 16} \, dx \).

\[
\begin{align*}
(1) & \quad \frac{x^2}{2} - 8x + C \\
(2) & \quad \frac{x^2}{2} - \frac{2}{5} \arctan \left( \frac{5x}{4} \right) + C \\
(3) & \quad \frac{x^2}{2} - \ln(25x^2 + 16) + C \\
(4) & \quad \frac{x^2}{2} - \frac{8}{5} \arctan \left( \frac{5x}{4} \right) + C
\end{align*}
\]

15. An appropriate substitution would transform the integral

\[
\int \frac{10}{x^3 \sqrt{x^2 - 25}} \, dx
\]

into which of the following?

\[
\begin{align*}
(1) & \quad \frac{2}{25} \int \cos^2 \theta \, d\theta \\
(2) & \quad \frac{2}{25} \int \sec^2 \theta \, d\theta \\
(3) & \quad \frac{2}{25} \int \cos^3 \theta \, d\theta \\
(4) & \quad \frac{2}{25} \int \tan \theta \sec^3 \theta \, d\theta
\end{align*}
\]