Instructions: please enter your NAME, ID NUMBER, FORM DESIGNATION, and your CRN on the op-scan sheet. The CRN should be written in the box labeled ‘COURSE’. Darken the appropriate circles below your ID number and below the Form designation letter. Use a number 2 pencil. Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1 through 17 of the op-scan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam.

1. Determine the value of \( \lim_{x \to 1} \sqrt{9 - x^2} \), if it exists.
   
   A. 0
   
   B. \( \sqrt{3} \)
   
   C. -\( \sqrt{3} \)
   
   D. The limit does not exist.

2. Which of the statements below is true about the functions \( f(x) = \frac{x^2 - 4}{x^2 - 9} \), \( g(x) = \begin{cases} x^2 - 3, & \text{if } x \leq 2 \\ 2x - 1, & \text{if } x > 2 \end{cases} \)?
   
   A. Only \( f \) is continuous at \( x=2 \).
   
   B. Only \( g \) is continuous at \( x=2 \).
   
   C. Both \( f \) and \( g \) are continuous at \( x=2 \).
   
   D. Neither function is continuous at \( x=2 \).

3. If \( f(x) = 2x^6 - 12x^4 \), then which one of the following is true?
   
   A. \( f \) has critical points only at \( x=0 \) and \( x=2 \).
   
   B. \( f \) has an inflection point only at \( x=0 \).
   
   C. \( f \) has a local maximum at \( x=-2 \).
   
   D. Concavity does not change at \( x=0 \).
4. Which of the following graphs satisfies all of the conditions 1 through 6?
   (1) \( f \) is continuous function on the interval \([0,3]\).
   (2) \( f'(1) = 0 \)
   (3) \( f'(x) > 0 \) for \( x < 1 \) and \( f'(x) < 0 \) for \( 1 < x < 2 \)
   (4) \( f'(2) \) does not exist
   (5) \( f''(x) < 0 \) for \( x < 2 \)
   (6) \( f''(x) < 0 \) for \( x > 2 \)

5. Differentiate \( y = x^{4x+1} \)

A. \( y' = x^{4x+1} \left( \ln(x^4 + 1 + 4) \right) \)

B. \( y' = 4 \ln(x + 1 + 4) \)

C. \( y' = x^{4x+1} \left( \frac{1}{x + 4} \right) \)

D. \( y' = 4x^{4x} \)

6. The Taylor Polynomial of degree three generated by \( f(x) = e^{3x} \) and centered at \( a = 0 \) is:

A. \( P_3(x) = 1 + 3x + 9x^2 + 27x^3 \)

B. \( P_3(x) = e^{3x} + 3e^{4x}x + 9e^{3x}x^2 + 27e^{3x}x^3 \)

C. \( P_3(x) = 1 + 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3 \)

D. \( P_3(x) = e^{3x} + 2e^{3x}x + 2e^{2x}x^2 + \frac{4}{3}e^{2x}x^3 \)
7. A water tank in the shape of a right circular cylinder with diameter 40 ft is being drained. If the level of the water decreases at $2 \frac{\text{ft}}{\text{min}}$, how fast is the water’s volume changing. Recall: $V = \pi r^2 h$.

A. The volume is decreasing at a rate of $40\pi h + 800\pi \frac{\text{ft}^3}{\text{min}}$.

B. The volume is decreasing at a rate of $800\pi \frac{\text{ft}^3}{\text{min}}$.

C. The volume is increasing at a rate of $800\pi \frac{\text{ft}^3}{\text{min}}$.

D. The volume is decreasing at a rate of $3200\pi \frac{\text{ft}^3}{\text{min}}$.

8. Which one of the following statements about $f(x) = \frac{2x^2 + 1}{x^2 - 1}$ is FALSE?

A. $f$ has a horizontal asymptote $y=2$.

B. $f$ has a vertical asymptote $x=-1$ and $x=1$.

C. $\lim_{x \to 1^-} f(x) \to \infty$ and $\lim_{x \to 1^+} f(x) \to \infty$

D. $f$ has a critical point at $x=0$.

9. Which of the following is the first iterate of Newton’s Method for the function $g(x) = \frac{e^{6x}}{x^5}$ with starting point $x_0=3$.

A. $\frac{36}{13}$

B. $\frac{-3}{13}$

C. $\frac{-6}{13}$

D. $\frac{42}{13}$
10. Which of the following is TRUE.

A. If \( f(x) \) is differentiable on \((a,b)\) & \(c\) is a point of a local maximum for \( f(x) \) in \((a,b)\), then \( f'(c) = 0 \).

B. If \( f'(x) = g'(x) \) for all \( x \) in the interval \( I \), then \( f(x) = g(x) \) on \( I \).

C. If \( f'(c) = 0 \), then \( f(x) \) has a maximum or a minimum value at \( x = c \).

D. If \( f(x) \) is continuous on \((a,b)\) then \( f(x) \) is differentiable on \((a,b)\).

11. If \( f(x) = 5x + \frac{3}{x} \) on the interval \([1,4]\), then what interior point \( x = c \) satisfies the conclusion of the Mean Value Theorem?

A. 1.79

B. 2.0

C. 3.92

D. The Mean Value Theorem does not apply to this function over this interval.

12. Consider the function \( f(x) = 1 - x^{\frac{5}{2}} \) on the interval \([-1,8]\). Which one of the following statements is true?

A. The absolute minimum of \( f \) is -1

B. The absolute minimum of \( f \) is \(-\frac{1}{3}\)

C. The absolute maximum of \( f \) is 0

D. The absolute maximum of \( f \) is 1

13. What is the slope of the tangent line to the curve \( x^3 + y^3 = 6xy \) at the point (3,3)?

A. \(-1\)

B. \(\frac{2}{3}\)

C. 1

D. \(-\frac{9}{7}\)
14. Differentiate: \( f(x) = \sqrt{4x^3 + 8x - 1} \)

A. \( f'(x) = \frac{1}{2\sqrt{12x^2 + 8}} \)

B. \( f'(x) = \frac{6x^2 + 4}{\sqrt{4x^3 + 8x - 1}} \)

C. \( f'(x) = \frac{1}{\sqrt{6x^2 + 4}} \)

D. \( f''(x) = \sqrt{12x^2 + 8} \)

15. The following three graphs, labeled a, b, and c, are the graphs of the function and its first and second derivatives. Match the appropriate letter to \( f(x), f'(x), f''(x) \).

A. \( c = f(x), \ a = f'(x), \ b = f''(x) \)

B. \( b = f(x), \ a = f'(x), \ c = f''(x) \)

C. \( b = f(x), \ c = f'(x), \ a = f''(x) \)

D. \( a = f(x), \ b = f'(x), \ c = f''(x) \)

16. Differentiate: \( f(x) = \frac{\sin(4x)}{x + 1} \)

A. \( f'(x) = 4\cos(4x) \)

B. \( f'(x) = \frac{4\cos(4x) - \sin(4x)}{(x + 1)} \)

C. \( f'(x) = \frac{\sin(4x) - 4(x + 1)\cos(4x)}{(x + 1)^2} \)

D. \( f'(x) = \frac{4(x + 1)\cos(4x) - \sin(4x)}{(x + 1)^2} \)
17. Differentiate: \( g(x) = \tan^{-1}(x^2 + x) \)

   A. \( g'(x) = -(2x + 1)\csc^2(x^2 + x) \)

   B. \( g'(x) = \frac{2x + 1}{1 + x^2 + 2x^3 + x^4} \)

   C. \( g'(x) = \frac{1}{1 + (x^2 + x)^2} \)

   D. \( g'(x) = \frac{2x + 1}{1 + x^2 + x^4} \)