1. Let $u(x,t)$ be a solution of $u_{tt} - u_{xx} = 0$ for $x \in (-\infty, \infty)$ and $t \in (-\infty, +\infty)$.

(a). If $u$ satisfies $u(x,0) = u_t(x,0) = 0$ for $x \in [0,1]$, find the region $D$ in $(x,t)$-plane such that $u(x,t) = 0$ for $(x,t) \in D$. Here $t$ can be negative.

(b). If $u(x,t)$ is known to vanish inside a triangle with vertices $(0,0), (1,1), (-1,-1)$ in $(x,t)$-plane. Give the largest domain in which you can conclude that the solution vanishes.

(Need to justify your answer)

2. Use the method of separation of variables to find two ordinary differential equations that will produce solutions $v(r,\theta)$ to the following partial differential equation.

$$\frac{\partial^3 v}{\partial r^3} + r^2 \frac{\partial^2 v}{\partial r^2} + r \frac{\partial v}{\partial r} + r^2 \frac{\partial^2 v}{\partial \theta^2} = 0.$$ 

Do not solve the ODEs.

3. Give a series solution for the following boundary-value problem.

$$u_{tt} = u_{xx}, \quad 0 < x < 1, \quad 0 < t,$$

$$u_x(0,t) = 0, \quad 0 < t,$$

$$u_x(1,t) = 0, \quad 0 < t,$$

$$u(x,0) = x, \quad 0 < x < 1,$$

$$u_t(x,0) = 1, \quad 0 < x < 1.$$

4. Give a series solution for the following boundary-value problem.

$$u_t = u_{xx}, \quad 0 < x < 1, \quad 0 < t,$$

$$u_x(0,t) = 0, \quad 0 < t,$$

$$u(1,t) = 0, \quad 0 < t,$$

$$u(x,0) = 3 - x, \quad 0 < x < 1.$$