Assignment Sheet # 3

Applied Partial Differential Equations

1. Show directly that the following problem is symmetric with respect to the usual inner product in $L^2(0,1)$ (i.e., show that the operator implied by the problem is symmetric).

$$-\phi'' = \lambda \phi, \quad \phi'(0) - \phi(0) = \phi(1) = 0.$$ 

2. Find all eigenvalues and eigenfunctions of the following problem. (If the eigenvalues cannot be found in closed form, indicate their values graphically.)

$$-y'' = \lambda y, \quad y(0) = y(1) - \frac{1}{3}y'(1) = 0.$$ 

3. By using the implicit function theorem, show that the system of equations

$$\begin{align*}
x^2 - yu &= 0 \\
xu + uv &= 0
\end{align*}$$

can be solved for $u$ and $v$ in terms of $x$ and $y$ in a neighborhood of the point $x = 4, y = 2, u = 8, v = -1.$