Homework 2
As part of the homework, email me all routines that you updated or wrote, so I can see how answers were obtained.

1. Use the programs `pdeldemo`, `nsoli`, `pdeleft`, `dxf`, and `dymf` to evaluate the scaling for solving the model PDE (partial differential equation) discussed in Chapter 3 of the Kelley book using the Poisson transform as a left preconditioner. Run `pdeldemo` for \( m = 31, 63, 127, 255 \) and make a table of the number of nonlinear iterations, the number of function evaluations, and the time (you have to put timers in the `pdeldemo`). Analyze how time scales with problem size (note the problem size is \( m^2 \))? First consider this as a theoretical question. What does the overall time depend on, taking into account the scaling of the number of function evaluations and/or the number of nonlinear iterations? Then compare with actual timings. Plot the relative residual (semilog plot) versus the number of nonlinear iterations and versus the number of function evaluations (already in the code) for \( m = 63 \).

2. Now adapt the program `pdeldemo` to evaluate the scaling for solving the model PDE in Chapter 3 without preconditioning. In the calls to `nsoli` replace `pdeleft` by `pde`. You also need to outcomment the solution using TFQMR and all related program statements, as TFQMR does not converge in the unpreconditioned case. Finally, in `pde.m` you need to uncomment the line for the unpreconditioned problem and outcomment the line for the preconditioned problem. If you have questions about adapting the matlab codes, come to office hours or bring a laptop to class. Run the updated (no preconditioning) version of `pdeldemo` for \( m = 31, 63, 127 \) and make a table of the number of nonlinear iterations, the number of function evaluations, and the time. What is the major cause of the time increase, more nonlinear iterations or more linear iterations (function evaluations)?

3. Adapt `pdeleft` and `pdeldemo` to solve the PDE

\[-(u_{xx} + u_{yy}) + 200u(u_x + u_y) = f,\]

where \( f \) is computed from the solution \( u = 10xy(1-x)(1-y)e^{x^4y^5} \), as is already done in the code. Solve the PDE preconditioned with the Poisson solver for \( m = 63 \) (plot results). Why is convergence so much slower?

4. Use the Chapter 4 routine `pdebrl` and related routines to compare Broyden’s method with the Newton-Krylov methods (with directional derivatives), for \( m = 31, 63, 127, 255 \). Which method is better in terms of nonlinear function evaluations, which method is better in time?

5. Write a short program that solves the following system of nonlinear equations

\[
\begin{align*}
1 - x_1 &= 0, \\
x_1x_2 - x_3x_4 &= 0, \\
1 - x_2^2 &= 0, \\
x_1 - x_2 &= 0,
\end{align*}
\]

using `nsoli` and GMRES with initial vector \( x = [0, 0, 0, 0]^T \).