Numerical Linear Algebra

Practical solution of linear algebra problems
→ efficient algorithms
→ properties of (numerical) solutions

NLA is of immense practical importance.
At the heart of most large-scale simulations, optimizations, analysis of systems,...
we find (very large) linear algebra problems.
(Often NLA part takes almost all the time)

Why NLA central to simulation, optimization?

Most important linear algebra problems:
- system of linear equations
- overdetermined system of linear equations
- underdetermined system of linear eq.s
  (with appropriate constraints/conditions)
- eigenvalue problems
- approximation of matrix functions

Why NLA central?

1) For more than a few vars, linear systems are the only ones we can solve directly. (finite number of calculations)
2) Many / most non-linear problems (most prob. in science & engineering) can be approximated (locally) by linear problems → solve non-linear problems by sequence of linear problems.

Non-linear (system of) partial diff. equations → (system linear) PDEs. Discretization gives (large) system of linear equations.

Alternatively, we can discretize first and solve system non-linear algebraic equations (sequence of linear problems).

3) Best approx. of (unknown / known) function in subspace → orthogonal projection

Very general principle with many applications!

(related to solving overdetermined system of equations)

4) Problems of vibrations, stability, resonance are related to eigenvalue problems.

(Local) linearization gives standard linear or generalized eigenvalue problem.

But more generally we can also consider non-linear or polynomial eigenvalue problems.
Eigenvectors and vectors also play an important role in the solution of very large systems of ODEs.

We will discuss much of this later, including applications.

Note that not only does NLA play an important role in solving (numerically) some linear algebra problem resulting from some approach to solve the original problem, but the linear algebra techniques/principles play fundamental role in deriving solutions (algorithms).
Overview

I. Linear systems of equations
   - direct solution for various types of problems
   - algorithms and cost
   - accuracy, sensitivity, stability
   - applications

II. Overdetermined and underdetermined problems
   - Overdetermined problems:
     x principles, least squares
     x solution, general principles
     x algorithms and cost
     x accuracy, sensitivity, stability
     x applications
     x singular value decomposition (SVD)
   - Underdetermined problems:
     x principles, min. norm solutions
     x other types of solution

III. Eigenvalue and singular value problems
   - definitions / basics
   - algorithms and analysis
   - algorithms for a few eigenpairs of large (sparse) matrices
   - algorithms for the SVD
   - accuracy, sensitivity, stability
   - generalized & polynomial eigenvalue problems
(Eigenvalue problems continued)
  - applications

IV Iterative methods for linear systems
  - fixed point iterations
  - Krylov subspace methods

V Advanced topics
  - nonlinear equations and optimization
  - model reduction