Name ______________________

Exam 1: Math 4245, Fall 2009. Due: 10 - 06 - 09.

1. Find the solution of the initial value problem

$$\frac{dy}{dx} = \frac{y - 2\,x}{y + 4\,x}, \qquad y(0) = 1. \qquad (\text{set } y(x) = x\,v(x))$$

2. Let p be a positive integer. Find the general solution of

$$x\,\frac{dy}{dx} + y = x^p + 1$$

by two different methods.

3. If $r(t)$ denotes the distance from a small mass to an asteroid with gravitational constant g, then $r(t)$ satisfies

$$\frac{d^2 r}{dt^2} = -\frac{g}{r^2}.$$

a) For $g = 1$, $r(0) = 10$, $\frac{dr}{dt}(0) = 0$, find the speed $\frac{dr}{dt}(T)$ for the value of T such that $r(T) = 1$.

b) In solution of a) we obtain a constant c and a first order initial value problem

$$\frac{dr}{dt} = z(r,\, c), \qquad r(0) = 10,$$

for a certain function $z(r,\, c)$. Solve this equation approximately, using Heun's method with $h = .25$ and use this approximate solution to estimate the numerical value of T described in a). (Use of Matlab, Mathematica, etc., recommended; programmable calculator OK. Show computations in detail.)

4. The opening angle, $\theta(t)$, of a screen door with closer satisfies

$$4\,\frac{d^2\theta}{dt^2} + \gamma\,\frac{d\theta}{dt} + \theta = 0.$$

Find γ such that the system is critically damped and, with this γ and $\theta(0) = \pi/2$, $\frac{d\theta}{dt}(0) = 0$, solve analytically (not numerically) and then, numerically, estimate T such that $\theta(T) = \pi/32$. Repeat with γ replaced by $2\,\gamma$. Is there any $\tilde{T}$ such that $\theta(\tilde{T}) = 0$?

5. With wind force on the door of Problem 4, 0 on the right hand side of the ODE is replaced by $-(1+t)^2$. Solve again with $\theta(0) = \pi/2$, $\frac{d\theta}{dt}(0) = 0$. Is there now $\tilde{T}$ such that $\theta(\tilde{T}) = 0$? Justify and, if $\tilde{T}$ exists, estimate it to three decimal places.