

## Exam 1 (in class); Math 3214

Be sure to put your name on each page you turn in.

1. Let  $A = (1 \ 3 \ 4)^*$ ,  $B = (2 \ -1 \ 1)^*$ . Compute

i)  $\langle A, B \rangle = B^*A$ ;    ii)  $A \times B - B \times A$ .

iii) Using standard matrix multiplication rules, compute  $AB^*$ .

2. Consider in  $R^3$  the parametric curve

$$\mathcal{C} : X(t) = (t + 1, \ t^2 + 2t + 2, \ 2t + 2)^*.$$

i) Compute  $X'(t)$ ,  $T(t)$  as vector functions of  $t$ .

ii) Give the formula for the tangent line to the curve  $\mathcal{C}$  at  $X_0 = X(0)$ .

iii) Compute the curvature,  $\kappa(0)$ , of  $\mathcal{C}$  at  $X_0$ .

3. Let  $\phi(x, y) = \cos(x+y) + \sin(x-y)$ . Compute the gradient vector and the Hessian matrix as functions of  $x, y$ . Identify the stationary points of  $\phi(x, y)$ .

4. Only one of the following matrices is positive definite. Which one? Justify your answer.

$$\mathbf{A} = \begin{pmatrix} 3 & -2 & -2 \\ -2 & 3 & -2 \\ -2 & -2 & 3 \end{pmatrix}; \quad \mathbf{B} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

5. Let  $f(x, y) = (x + y)^3 - 3(x + y) + (x - y)^2$ . Find all stationary points and classify them as local maxima, local minima, or neither.