Use this sheet as a cover sheet for your exam paper. Staple, please.

Name ______________________

Exam 2: Math 2224, Fall, 2009; Due: 11/06/2009.

1. Let $f(x,y) = \sqrt{9\,x^2 + 4y^2 - 11}$. Describe precisely which points (x,y) lie in the domain of $f(x,y)$ and which numbers lie in the range of $f(x,y)$.

2. For $f(x,y)$ as in **1.** sketch the contour curve $\mathcal{C}(f) = \{(x,y)\,|f(x,y) = 5\}$.

3. For each of the following, determine **i)** if $lim_{(x,y) \to (0,0)} f(x,y)$ exists; **ii)** if $f(x,y)$ is continuous at $(0,0)$; **iii)** if $f(x,y)$ is continuously differentiable at $(0,0)$. Justify your conclusions in each case.

a) $f(x,y) = \frac{x^4-y^4}{x^4+y^4}$; **b)** $f(x,y) = \frac{x^2-y^2}{|x|+|y|}$, $(x,y) \neq (0,0)$, $f(0,0) = 0$.
c) $f(x,y) = \frac{x^2y^2}{x^2+y^2}$, $(x,y) \neq (0,0)$, $f(0,0) = 0$.

4. Let $f(x,y)$ be defined as in Problem 1. Compute the linear approximation to $f(x,y)$ at the point $(x_0, y_0) = (1,2)$. Use that approximation to estimate $f(.95, 2.05)$ and compare with the actual value $f(.95, 2.05)$.

5. A roller-coaster track traces out the curve $X(t) = (x(t)\,y(t)\,z(t))^*$ in R^3, with $x(t) = 2\,\cos t$, $y(t) = 3\,\sin t$, $z(t) = g(x(t), y(t))$, where $g(x,y) = (x+y)^2 - (x-y)^2$. Use the Chain Rule to find the slope of the track at the point corresponding to $t = 3\pi/4$. Where is the highest point on the track and how high is it at that point?

6. Find a and b for which the line $y = ax + b$ provides the least squares linear fit to $y_k = \sin(x_k) + \frac{1}{2}\cos(x_k)$, $k = 1, 2, 3, 4, 5$, at the points $x_1 = 0$, $x_2 = \pi/6$, $x_3 = \pi/4$, $x_4 = \pi/3$ and $x_5 = \pi/2$. Verify that the values of a and b which you obtain are a global minimum for

$$\ell(a,\,b) = \sum_{k=1}^{5} (a\,x_k + b - y_k)^2 .$$

Sketch, on the same graph, the points $(x_k,\, y_k)$ and the line $y = a\,x + b$.

7. List all of the critical points, i.e., points where the gradient equals 0, for the function $f(x,y) = (x^3 - 12\,x)\,(y^3 - 27\,y)$. Find at least one of these which is a local maximum for $f(x,\,y)$.