Name _____

Exam 1: Math 2224; Due: 9-28-09

1. Determine whether the *sequence* (not the *series*!) converges or diverges as $n \to \infty$. If it converges, find the limit. Justify your answers in all cases.

$$a)\ a_n = \frac{3+5n^2}{n+n^2}; \quad b)\ a_n = \ln(n+1) - \ln(n);$$

$$c)\ a_n = \frac{\tan(1/n)}{\sin(2/n)}; \quad d)\ a_n = \frac{\cot(1/n)}{\cos(2/n)}.$$

2. Determine whether the given *series* is convergent or divergent, carefully justifying your answer in each case. If the series converges, find its sum.

$$a)\ \sum_{n=1}^{\infty} \frac{3^n - 2^n}{6^n}; \quad b)\ \sum_{n=0}^{\infty} \frac{\cos(n\,\pi/4)}{2^n}; \quad c)\ \sum_{n=2}^{\infty} \frac{1}{n\,\log(n)}.$$

3. Determine whether the series is convergent or divergent. Justify your answer.

$$a)\ \sum_{n=1}^{\infty} \frac{n}{n^2+1}; \quad b)\ \sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{1+n^2}; \quad c)\ \sum_{n=1}^{\infty} \frac{\sin(n\,\pi/4)}{n^2 - 3\,n + 10}.$$

4. Find a power series representation for the given function $f(x)$, about the given center x_0, and determine the radius and interval of convergence.

$$a)\ f(x) = \frac{1}{1+9x^2},\ x_0 = 0; \quad b)\ f(x) = \frac{x}{1-x},\ x_0 = 3;$$

$$c)\ f(x) = \frac{3}{x^2 - x - 2},\ x_0 = 0.$$

Hint: For c), use the partial fractions decomposition procedure to write $f(x)$ in the form $f(x) = \frac{c_1}{x-2} + \frac{c_2}{x+1}$ for appropriate constants $c_1,\ c_2$.

5. Find an expression for the coefficients c_k in the Mclaurin series $\sum_{k=0}^{\infty} c_k\,x^k$ for

$$a)\ f(x) = \log(x+2); \quad b)\ g(x) = \frac{1}{(x+2)^2}.$$

6. A parallelopipedon $P \subset R^3$ has sides (i.e., edges) corresponding to the vectors

$$A = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad B = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, \quad C = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}.$$

Find the volume of P and the area of its surface. What is the greatest distance between any two points in P?