Name ______________________________

Exam 2: Math 3214

In-class part:

1. Let $\phi(x,\, y) \;=\; y\,\sin\pi x \,+\, x\,\cos\pi y$ and let $\mathcal{C}$ be the curve described parametrically by $x \;=\; t,\;\; y \;=\; t^2,\;\; 0 \;\leq\; t \;\leq\; 1$. With $F(X) \;=\; F(x,\, y) \;=\; \nabla\phi(x,\, y)$, the gradient of $\phi(x,\, y)$, compute the line integral $\int_{\mathcal{C}} F(X)^* dX$.

2. Let $\mathcal{R}$ be the region $\mathcal{R} \;=\; \{(x, y)\;|0 \;\leq\; y \;\leq\; 1 - (x-1)^2,\;\; 0 \;\leq\; x \;\leq\; 2\}$, and let $\mathcal{C}$ be the positively oriented boundary of $\mathcal{R}$. Let

$$G(X) \;=\; \begin{pmatrix} (x-1)^2 \\ y^2 \end{pmatrix} \;=\; (x-1)^2\,\mathbf{i} \,+\, y^2\,\mathbf{j}.$$

For $X \,\in\, \mathcal{C}$, $\nu(X)$ is the unit exterior normal to $\mathcal{C}$ at that point.

a) Compute the integral $\int_{\mathcal{C}} G(X)^*\nu(X)\,ds$ by using the divergence theorem.
b) Compute the line integral $\int_{\mathcal{C}} G(X)^* dX$ by using Green's theorem.

3. Find a scalar valued function $\phi(X) \;=\; \phi(x,\, y)$ such that $H(X) \;=\; \nabla\phi(X)$ is the gradient of $\phi(x,\, y)$, where $H(X)$ is the vector field

$$H(X) \;=\; H(x,\, y) \;=\; \begin{pmatrix} 2xy + y^2 \\ x^2 + 2xy \end{pmatrix}.$$

4. Let $H(X)$ be the vector field in **3.** Compute the Jacobian matrix $\nabla H(X)$ as a function of x and y. Using the linear approximation to $H(X) \;=\; H(x,\, y)$ at $x \;=\; 1,\;\; y \;=\; 2$, estimate $H(1.2, 2.1)$.

5. Let $\mathcal{B}$ be the ball of radius 1, centered at the origin, in R^3. Compute the integral

$$\iiint_{\mathcal{B}} \left(x^2 + y^2 + z^2\right)^{3/2}\, dx\, dy\, dz.$$

6. The equation $x \,+\, e^{\,x} \;=\; 4$ has a solution $\hat{x}$ in the interval $1 \;\leq\; x \;\leq\; 2$. The equation can be written in "fixed point" form as

$$\text{a)}\quad x \;=\; 4 \,-\, e^{\,x}, \qquad \text{or} \qquad \text{b)}\quad x \;=\; \log\left(4 \,-\, x\right).$$

Indicate which form you would use to solve the equation by fixed point iteration and justify your choice.