

COMMENTS

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Comment on "A numerical study of periodic disturbances on two-layer Couette flow" [Phys. Fluids 10, 3056 (1998)]

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The flow of fluids with different viscosities, subjected to an interfacial perturbation, can lead to fingering and migration.¹ Both Refs. 2 and 3 refer to fingers in two-dimensional studies, which would correspond to sheets in three dimensions. Figure 19 of Ref. 3 shows a sequence of interface positions for initial amplitude 0.05, wave number $\alpha = \pi/2$, Reynolds number $Re_1 = 500$, viscosity ratio $m = 0.5$, undisturbed interface height $l_1 = 0.372$, and interfacial tension parameter $T = 0.01$, at times 0, 5, 10, 15, and 20 s, carried out on a 256×256 mesh. A region of high curvature forms, followed by pinching into a series of horizontal drops. However, the drops are at the scale of the numerical mesh. A spatial convergence test is reported here, showing that with a refined mesh, the drops are replaced by an elongated finger.

The wave number corresponds to the largest growth rate mode according to linearized stability theory of two-layer Couette flow. The computation for initial amplitude 0.05 is performed on a refined mesh, 512×512 , in Fig. 1. The lower fluid forms a short stubby finger into the upper more viscous liquid, and the upper fluid forms a narrow sharply pointed finger into the lower liquid. The evolution up to time 10 s is identical for both the 256 and 512 meshes. The difference arises in the first detachment of a drop and for features which are on the scale of the numerical cell. For the 256×256 mesh, the first drop is detached from the lower finger at time 12 s, while in the 512×512 mesh this occurs at 13 s (Fig. 1), and for the 1024×1024 case at 15 s. The mesh refinement study indicates that these drops are numerical and not physi-

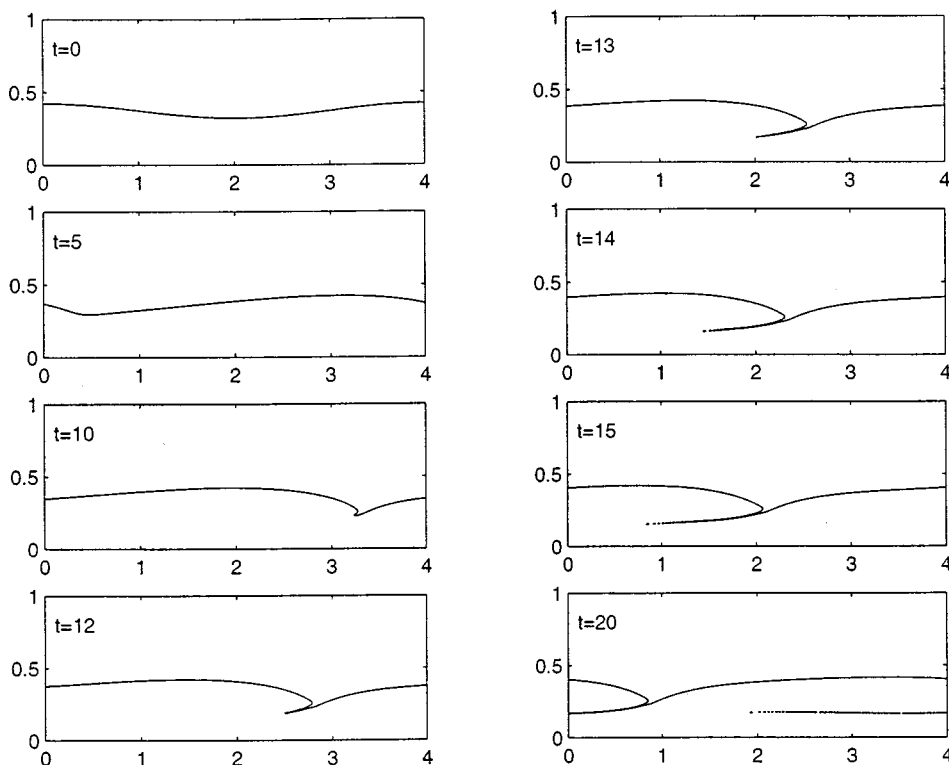


FIG. 1. Sequence of interface positions for $A(0) = 0.05$, $\alpha = \pi/2$, $Re_1 = 500$, $m = 0.5$, $l_1 = 0.372$, $T = 0.01$, equal densities, and zero gravity. The calculation is carried out on a 512×512 mesh.

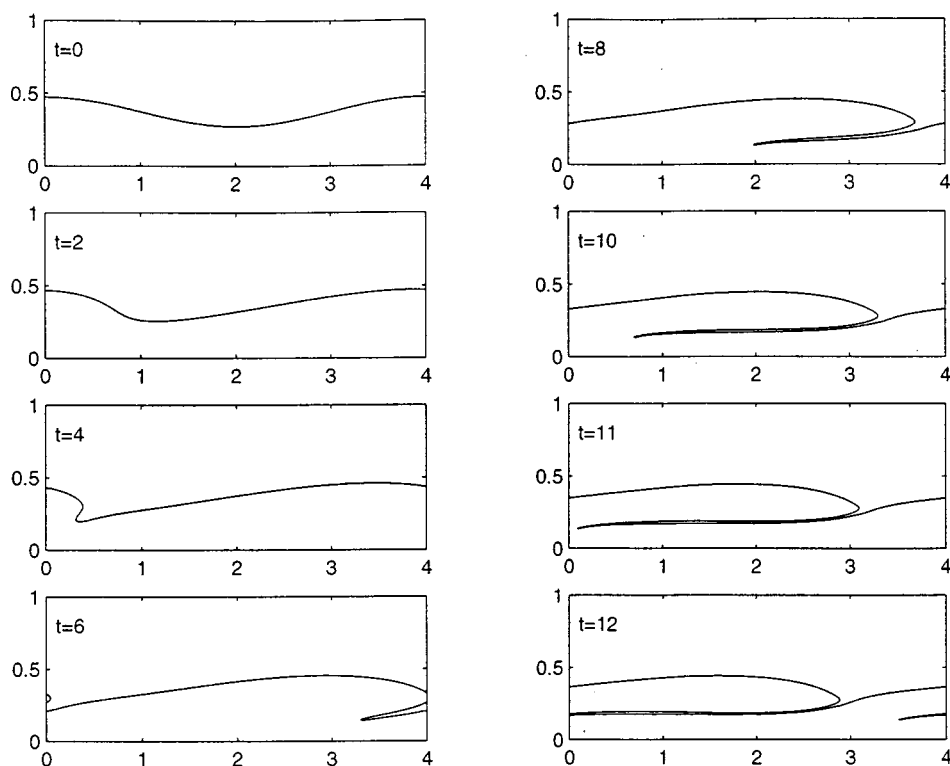


FIG. 2. Interface positions at $t=0, 2, 4, 6, 8, 10, 11, 12$ s for $A(0)=0.1$, $\alpha = \pi/2$, $Re_1=500$, $m=0.5$, $l_1=0.372$, $T=0.01$, equal densities, and zero gravity. The calculation is carried out on a 512×512 mesh.

cal; for example, the 1024 by 1024 mesh at 20 s shows an elongated finger ending in several drops, rather than the numerous small drops of Fig. 1 on the 512×512 mesh. The features of the flow that are much larger than the numerical cell dimension, such as the shape of the finger, are mesh-converged at the 256 or 512 mesh level. A larger initial amplitude illustrates this in Fig. 2. We conclude that we have elongated finger formation or sheet formation rather than the tip-streaming picture which was previously observed on the less refined mesh of Ref. 3. When the initial amplitude is decreased to $A(0)=0.001$, as shown in Fig. 18 of Ref. 3, the sequence of events occurs at a later time, with finger exten-

sion occurring around time $t=120$ s. When the initial amplitude is increased to $A(0)=0.1$, the interface evolves in a qualitatively similar manner but sooner.

ACKNOWLEDGMENT

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¹D. D. Joseph and Y. Y. Renardy, *Fundamentals of Two-Fluid Dynamics* (Springer-Verlag, New York, 1993).

²C. Pozrikidis, "Instability of two-layer creeping flow in a channel with parallel-sided walls," *J. Fluid Mech.* **351**, 139 (1997).

³J. Li, Y. Renardy, and M. Renardy, "A numerical study of periodic disturbances on two-layer Couette flow," *Phys. Fluids* **10**, 3056 (1998).