Total Change Theorem

The integral of a rate of change is the total change from a to b.

\[ \int_{a}^{b} F'(x) \, dx = F(b) - F(a) \]

Ex. \[ \int_{a}^{b} v(t) \, dt = \]

Note units: \( v(t) = \text{ft/sec} \quad \text{dt = sec, so} \)

\[ \int_{a}^{b} v(t) \, dt = \left( \frac{\text{ft}}{\text{sec}} \right) (\text{sec}) = \text{ft} \]
Distance =

Displacement =

\[ \int_{a}^{b} f(t) \, dt = \]

Ex. Find the displacement and the distance traveled by a particle whose velocity is measured by \( v(t) = t^2 - 2t - 8 \quad 1 \leq t \leq 6 \)

Displacement:
Distance:

Area

\[ f(x) \]

\[ f(y) \]
Ex. Find the area of the region bounded by the x-axis, y-axis and 
y = 2 – 2x.

Ex. Find the area of the region bounded by the y-axis and the 
curve \( x = 2y + 3y^4 - 2y^6 \).
Do: 1. Find the area of the region bounded by the y-axis, $y = 1$, and $x = y^4$ using $y$ as your variable.

2. Let $v(t) = 2 - t$, $0 \leq t \leq 3$. Find the displacement and distance traveled.
Piecewise Functions:

Ex. Let \( C = \int_0^4 g(y) \, dy \) where

\[
g(y) = y^2 - 15 \quad \text{when} \quad y \leq 3 \quad \text{and} \quad g(y) = -2y \quad \text{when} \quad y \geq 3
\]

Determine which of the following holds:

a. \( C \leq -10 \)

b. \(-10 < C < -3 \)

c. \(-3 < C < 0 \)

d. \(0 < C < 3 \)

e. \(3 < C < 10 \)

f. \(10 < C \)

Do: Find \( C = \int_0^2 h(x) \, dx \) when

\[
h(x) = x \quad \text{when} \quad x \leq 1 \quad \text{and} \quad h(x) = x^2 \quad \text{when} \quad x \geq 1
\]