2.4b Supplementary Problems

NAME: ________________  CRN: ________

• In problems 1-3, use the graph of $y = f(x)$ to answer the questions.

1. $y$

   100  200  300  400

   1  2  3  4

(a) Does there exist a value of $\delta > 0$ such that if $0 < |x - 2| < \delta$ then $f(x) > 100$? Justify your answer.

(b) Does there exist a value of $\delta > 0$ such that if $0 < |x - 2| < \delta$ then $f(x) > 300$? Justify your answer.

(c) If a value of $\delta$ exists for both part (a) and (b), which value of $\delta$ do you expect to be smaller?
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3.

(a) Does there exist a value of $\delta > 0$ such that if $0 < |x - 2| < \delta$ then $f(x) > 100$? Justify your answer.

(b) Does there exist a value of $\delta > 0$ such that if $0 < |x - 2| < \delta$ then $f(x) > 300$? Justify your answer.

(c) If a value of $\delta$ exists for both part (a) and (b), which value of $\delta$ do you expect to be smaller?
4. (a) Consider the function \( g(x) = \frac{1}{x^4} \). Explain briefly why for each \( M > 0 \), there exists a \( \delta > 0 \) such that if \( 0 < |x| < \delta \) then \( \frac{1}{x^4} > M \).

(b) Consider the function \( f(x) = \sin \left( \frac{1}{x^4} \right) \).

(i) Does there exist a \( \delta > 0 \) such that if \( 0 < |x| < \delta \) then \( f(x) > 4 \)? Justify your answer.

(ii) Does there exist a \( \delta > 0 \) such that if \( 0 < |x| < \delta \) then \( f(x) > \frac{1}{2} \)? Justify your answer.