Math 2534 Solution Homework 3 Chapter 3

Problem 1:
Put the following sentences into symbolic logic using single quantifiers. Define your variables, the domain and the predicate.

Part A:

a) Most children do not like green peas.
   Define Domain C to be set of all children and P(x) = x like peas.
   $\exists x [x \in C \land \neg P(x)]$

b) No student likes tests.
   Define Domain S to be set of all students and P(x) = x likes test.
   $\forall x, x \in S \rightarrow \neg P(x)$

c) Each of us have a secret.
   Define Domain P to be set of all people and S(x) = x has a secret.
   $\forall x, x \in P \rightarrow S(x)$

d) Hardly anybody can spell correctly.
   Define Domain P to be set of all people and S(x) = x can spell correctly.
   $\exists x [x \in P \land S(x)]$

Part B: Show all work

1) Negate the symbolic logic for a) and then put into a conversational English sentence.
   $\neg [\exists x [x \in C \land P(x)]] \equiv \forall x, \neg ([x \in C] \lor \neg P(x)) \equiv \forall x, x \in C \rightarrow \neg P(x)$
   No Child likes peas.

2) Negate the symbolic logic for b) and then put into a conversational English sentence.
   $\neg [\forall x, x \in S \rightarrow \neg P(x)] \equiv [\forall x, \neg ([x \in S] \lor \neg P(x))] \equiv \exists x \mid \neg ([x \in S] \land \neg P(x))$
   There is at least one student that likes tests.
Problem 2:
Put the following sentences into symbolic logic using multiple quantifiers. Define your variables, the domains and the predicate.

a) All of us must obey some rules.
Let the domain P be set of all people (Let x be an arbitrary person.)
Let the domain R be the set of all rules (Let y be an arbitrary rule.)
B(x,y) = x obeys y.
\( \forall x, x \in P \rightarrow [\exists y | y \in R \land B(x, y)] \)

b) Hardly any of us listen to any news programs.
Let the domain P be set of all people (Let x be an arbitrary person.)
Let the domain N be the set of all news program (Let y be an arbitrary news program.)
W(x,y) = x listens to y.
\( \exists x | x \in P \land [\forall y, y \in N \rightarrow W(x, y)] \)

c) There is a national park we would all love to visit.
Let the domain P be set of all people (Let x be an arbitrary person.)
Let the domain N be the set of all National parks. (Let y be an arbitrary National park.)
K(x, y) = x loves to visit y.  \textbf{Correction}
\( \exists y | y \in N \land [\forall x, x \in P \rightarrow K(x, y)] \)
**Problem 3:**
Given the following domains and predicate, put each symbolic statement into natural conversational English.

Domain B: all birds \( (x = \text{an arbitrary bird}) \)
Domain F: all types of fruit \( (y = \text{arbitrary type of fruit}) \)
Predicate: \( E(x,y) = \text{x will eat y} \).

1) \( \forall x \in B, \exists y \in F \big| E(x,y) \)
Each bird has a type of fruit they will eat.

2) \( \exists y \in F \big| \forall x \in B, E(x,y) \)
There exist one type of fruit that all birds will eat.

3) \( \exists x \in B \big| \forall y \in F, E(x,y) \)
There is a bird that will eat all types of fruit.

4) \( \forall y \in F, \exists x \in B \big| E(x,y) \)
For each type of fruit there is a bird that will eat it.

**Problem 4:**
A discrete math class contains 10 CS majors who are a freshman, 32 engineering majors who are sophomores, 50 CS majors who are sophomores, 5 engineering majors who are juniors, and 6 CS majors who are juniors, and one engineering major who is a senior. There are no double majors. Determine the truth sets for each of the statements below. (ie. The set of elements that will make the statement true) or give a counter example.

1) There is a student in the class who is not a junior.
2) There is a student in the class who is neither a CS major nor a sophomore.
3) There is a student in the class who is freshman and not a CS major.
4) There is a student in the class who is a sophomore and an engineering major.
5) There is a student in the class who is engineering major and a senior.

Solution:

1) TS = {10 CS-F, 32E-Sp, 50CS-Sp, 1E-S}
2) TS = {5E-J, 1E-S}
3) TS = null set (no members)
4) TS = {32E-Sp}
5) TS = {1E-S}
**Problem 5:** Prove the following using the definitions of even and odd.

**Theorem:** Given odd integers $a$ and $b$, $3b + a^2$ is even.

Proof: Given that $a$ and $b$ are each odd, by definition of odd there exist integers $k$ and $p$ so that $a = 2k + 1$ and $b = 2p + 1$.

Now consider:

$$3b + a^2 = 3(2p + 1) + (2k + 1)^2 = 6p + 3 + 4k^2 + 4k + 1$$

$$= 6p + 4k^2 + 4k + 4 = 2(3p + 2k^2 + 2k + 2) = 2m$$

where $m = 3p + 2k^2 + 2k + 2$ is an integer.

Therefore $3b + a^2 = 2m$ is even by definition of even.