Math 2534  Sol Homework 3  Fall 2015

Problem 1: Put the following sentences in symbolic logic using a quantifier. Define the domain and the single variable predicate for each statement

a) Most students read a book.
   Let S be the set of all students (x)
   \( P(x) = x \) reads a book.
   \[ \exists x \in S \quad \neg P(x) \]

b) Every child likes dinosaurs.
   Let C be the set of all children (x)
   \( D(x) = x \) likes dinosaurs.
   \[ \forall x, x \in C \rightarrow D(x) \]

c) Not all CS majors minor in math.
   Let CS be the set of all CS majors (x)
   \( M(x) = x \) has a minor in math.
   \[ \exists x \in S \quad \neg M(x) \]

Problem 2: Convert the logic statement into natural conversational English

a) Domain D: all professors
   Predicate \( O(x) = x \) holds office hours.
   \[ \forall x, x \in D \rightarrow O(x) \]
   All Professors hold office hours

b) Domain B: all VT students
   Predicate \( H(x) = x \) does homework.
   \[ \exists x \in B \quad \neg H(x) \]
   There is a student that does not do homework.

Problem 3: Put into symbolic logic with multiple quantifiers and define the domains and the multi-variable predicate for each statement.

a) There are some books that all students must read.
   Let B be the set of all books (x).
   Let S be the set of all students (y).
   \( R(x,y) = x \) reads y.
   \[ \exists x \in B \quad [\forall y, y \in S \rightarrow R(x,y)] \]

b) Any student likes some sport.
   Let S be the set of all students (x).
   Let P be the set of all sports (y).
   \( L(x,y) = x \) likes y.
   \[ \forall x, x \in S \quad [\exists y \in P \wedge L(x,y)] \]
Problem 4:
Given the following domains and predicate, put each of the following into conversational English.

**Domain D:** all marching bands \((x = \text{an arbitrary band})\)

**Domain B:** all half-time shows \((y = \text{an arbitrary half time show})\)

**Predicate:** \(P(x,y) = \text{x preforms in y.}\)

\[\exists x \in D \mid \forall y \in B, P(x,y)\]

There is a band that preforms in all half-time shows.

\[\forall y \in B, \exists x \in D \mid P(x,y)\]

Each half-time show has at least one band which preforms.

\[\exists y \in B \mid \forall x \in D, P(x,y)\]

There is a half-time show in which all bands preform.

\[\forall x \in D, \exists y \in B \mid P(x,y)\]

Each band preforms in at least one half-time show.

**Problem 5:** A discrete math class contains 2 CS majors who are a freshman, 15 engineering majors who are sophomores, 50 CS majors who are sophomores, 5 engineering majors who are juniors, and 10 CS majors who are juniors, and no senior engineering majors or CS majors. There are no double majors. Determine the truth sets for each of the statements below. (ie. The set of elements that will make the statement true—there also a possibility that the truth set is an empty set)

1) There is a student in the class who is not a junior.
   \(TS = \{2\text{CS-F}, 15\text{ENG-S}, 50\text{CS-S}\}\)

2) There is a student in the class who is neither a CS major nor a sophomore.
   \(TS = \{5\text{ENG-J}\}\)

3) There is a student in the class who is freshman and not a CS major.
   \(TS = \{\text{empty set}\} = \emptyset\)

4) There is a student in the class who is a sophomore and an engineering major.
   \(TS = \{15\text{ENG-S}\}\)

5) There is a student in the class who is engineering major and a senior.
   \(TS = \{\text{empty set}\} = \emptyset\)

**Problem 6:** Prove directly or give a counter example for the following theorem.

**Theorem 1:** If \(a\) and \(b\) are odd integers and \(c\) is an even integer, then \((a + c)^2 + b^2\) is even.

**Proof:** We are given that \(a\) and \(b\) are each odd and \(c\) is even. We have that \(a + c\) is odd since the sum of an odd and even integer is always odd. We also have \((a + c)^2 = (a + b)(a + b)\) is also odd since the product of two odd integers is odd. It is also true that \(b^2 = bb\) is odd since the product of two integers is odd. Now we have that \((a + c)^2 + b^2\) is even since the sum of two odd integers is even.