Math 2534 Sol Homework 2  Sec 2.1-2.3

Problem 1: Use Algebra of Logic to Prove the following:

Theorem : \[ (\sim (\sim(p \rightarrow q)) \lor (\sim p \land \sim q)) \rightarrow (p \land q) \equiv p \]

Proof:
\[
\begin{align*}
[\sim (\sim(p \rightarrow q)) \lor (\sim p \land \sim q)] & \rightarrow (p \land q) \equiv & \text{given} \\
\sim[\sim (\sim(p \rightarrow q)) \lor (\sim p \land \sim q)] & \lor (p \land q) \equiv & \text{Implication Law} \\
\sim[\sim (p \lor q) \lor (\sim p \land \sim q)] & \lor (p \land q) \equiv & \text{Double Negative Law} \\
[\sim (p \lor q) \land (\sim p \land \sim q)] & \lor (p \land q) \equiv & \text{DeMorgan's Law} \\
[(p \lor q) \land (p \land q)] & \lor (p \land q) \equiv & \text{Double Negative Law} \\
[p \lor (q \land q)] & \lor (p \land q) \equiv & \text{DeMorgan's Law} \\
[p \lor (p \land q)] & \equiv & \text{Double Negative Law} \\
[p \lor (p \land q)] & \equiv & \text{Distributive Law} \\
[p \lor (p \land q)] & \equiv & \text{Distributive Law} \\
[p \lor (p \land q)] & \equiv & \text{Identity Law} \\
p & \equiv & \text{Absorption Law}
\end{align*}
\]

Therefore \[ \sim (\sim(p \rightarrow q)) \lor (\sim p \land \sim q)] \rightarrow (p \land q) \equiv p \]

Problem 2: Put the following symbolic implication form. Define all your variables.

a) Only if you come to class and do the homework will you pass this course.
   Let Y be the statement “you come to class”
   Let H be the statement “you do the homework”
   Let P be the statement “you pass the course”
   \[ P \rightarrow (Y \land H) \]

b) You do not drink or you will not drive.
   Let R be the statement “you drink”
   Let D be the statement “you drive”
   \[ \sim R \lor \sim D \equiv R \rightarrow \sim D \]

c) I will go to the party if you do.
   Let Y be the statement “you go to the party”
   Let I be the statement “I go to the party”
   \[ Y \rightarrow I \]
Problem 3: Are any the following statements equivalent? Put into symbolic logic and justify your reasoning by comparing sufficient and necessary conditions.

Let Y be the statement “you dance”
Let I be the statement “I dance”

a) I will dance or you do not dance. \( I \lor \neg Y \equiv \neg I \rightarrow \neg Y \equiv Y \rightarrow I \) (contrapositive)

b) I will dance, if you dance. \( Y \rightarrow I \)

c) Only if you do not dance, will I not dance. \( \neg I \rightarrow \neg Y \equiv Y \rightarrow I \) (contrapositive)

d) I do not dance and you dance. \( \neg I \land Y \equiv \neg (I \lor \neg Y) \) (contradiction)

The statements a), b), and c) are equivalent since they have the same sufficient and necessary condition.

Problem 4: Given the statement: If the door is open then the professor is having office hours.

a) Put this statement into symbolic logic notation and define your variables.

b) Negate the symbolic logic in part a) and then put into an English sentence.

Solution: Let D represent the statement “the door is open”
Let O represent the statement “the professor is having office hours”
\( D \rightarrow O \)

Negate:
\( \neg [D \rightarrow O] \equiv \neg (\neg D \lor O) \equiv D \land \neg O \)

The door is open but the professor is not in his office.

Problem 5: Determine if the following arguments are valid and justify your conclusion.

Put each argument into symbolic logic and define all variables. In justifying your conclusion be sure to indicate what is the sufficient condition and what is the necessary condition.

a) If you miss practice, you will not be in the band.
   You are not in the band.
   Therefore you did miss practice

Let P be the statement “you miss practice”
Let B be the statement “you miss practice”
\( P \rightarrow \neg B \)
\( \neg B \)
\( \therefore P \)

Since we only have the necessary condition we can not guarantee the sufficient condition. This is Converse Error and argument is not valid.
b) If the weather is bad, the game will be canceled.
   The weather was good.
   Therefore the game was held.

Let \( W \) be the statement "Weather is good"
Let \( G \) be the statement "the Game is held"
\[
\sim W \rightarrow \sim G
\]
\[
W
\]
\[
\therefore G
\]
This is the inverse error. Argument is not valid

c) If you bake your famous apple pie, you will win first place.
   You did not win first place.
   Therefore you did not bake your pie.

Let \( P \) be the statement "you bake your apple pie"
Let \( F \) be the statement "you will win first prize"
\[
P \rightarrow F
\]
\[
\sim F
\]
\[
\therefore \sim P
\]
This argument is valid by the Contrapositive.

Problem 6: For each of the following sets of premises, determine the implied conclusion and justify your reasoning. Use symbolic logic and define all variables.

a) If it is October we will go see the fall leaves. We did not go see the leaves.
   Therefore it is not October (by the contrapositive)

b) Only if you go will I go. I did go. Therefore you did go. (Since “I did go” is the sufficient condition and guarantees the necessary condition.)

c) You will take the train if I do. You did not take the train. Therefore I did not go. (by the contrapositive)
Problem 7
Puzzle using Conditional logic.

Three siblings Alice, Bob and Carol truthfully reported their grades to their parents as follows:

Alice: If I passed math, then so did Bob.
I passed English if and only if Carol did.

Bob: I passed math only if Alice did.
Alice did not pass History.

Carol: Either Alice passed history or I did not pass it.
If Bob did not pass English, then neither did Alice.

If each of the three passed at least one subject and each subject was passed by at least one of
the three, and if Carol did not pass the same number of subjects as either of her siblings, which
subjects did they each pass?

Define your variables as follows:

$A_E$ means that Alice passed English
$A_M$ means that Alice passed Math
$A_H$ means that Alice passed History
$B_E$ means that Bob passed English
$B_M$ means that Bob passed Math
$B_H$ means that Bob passed History
$C_E$ means that Carol passed English
$C_M$ means that Carol passed Math
$C_H$ means that Carol passed History

Solution:
Alice passed Math and English. Bob passed all three subjects. Carol passed English.

Bob states that Alice did not pass history. Carol states “Either Alice passed history or I did
not pass it.” Since we already know that Alice did not pass history, then the statement that She
did pass history is false. So the Exclusive Or indicates that Carol’s statement “I did not pass it
must be true. So Carol did not pass history. Since every subject was passed by at least one
person, Bob passed history.
Now consider Carol’s statement “If Bob did not pass English, then neither did Alice.” And Alice’s statement “I passed English if and only if Carol did.” If it were true that Bob did not pass English, then this sufficient condition guarantees that Alice did not pass English as well. If Alice did not pass then by the “if and only if” statement we know that Carol also did not pass. (since the two statements must have the same truth value of false.) However this means that no body passed English. But very subject was passed by at least one person, so this means that everyone passed English.

Alice states that “If I passed math, then so did Bob.” And Bob states that “I passed math only if Alice did.” This is equivalent to stating that “Alice passes Math if and only if Bob passes Math.” Suppose that Alice and Bob do not pass math. Then Carol must have passed math since every course is passed by at least one person. This means that Carol will pass two classes, Alice passes one and Bob passes two. But Carol does not pass the same number of subjects as either of her siblings. Therefore Alice and Bob did pass math and Carol did not.