Math 2534     Homework 9     PMI and Sets

Show all work.

Problem 1: (Use PMI)

Theorem: Given the sequence function \( f(n) = 3^n - 2^n \) and the recursive sequence
\[ a_1 = 1, \quad a_2 = 5, \quad a_n = 5a_{n-1} - 6a_{n-2}, \quad n > 2, \] Then \( f(n) = a_n \) for all natural numbers.

Problem 2: (direct proof)

Theorem: Given the Fibonacci sequence \( f_n \), then \( f_{n+2} - f_{n+1} = f_n f_{n+3}, \forall n \in N \)

Problem 3: Given sets \( A = \{a, b, \{c\}, c\}, B = \{a, \{b, c\}, d, \emptyset\}, C = \{b, c\} \)

Find the following: (don’t forget to use equal signs.)

a) Find the following sets:
   1) \( A \cap B \)
   2) \( B \cup C \)
   3) \( B - A \)
   4) \( B \cap C \)
   5) \( A - C \)

Problem 4: Given sets \( A = \{a, b, \{c\}, c\}, B = \{a, \{b, c\}, d, \emptyset\}, C = \{b, c\} \)

Indicate if the following is true or false. (do not justify)

1) \( \emptyset \in C \)
2) \( \emptyset \subset B \)
3) \( \{\emptyset\} \subset A \)
4) \( \emptyset \subset A \)
5) \( c \in B \)
6) \( C \subset B \)
7) \( \{a, b\} \in P(A) \)
8) \( \{\{c\}\} \subset A \)
9) \( \{c\} \in A \)