Math 2534  Homework 8  PMI

Prove the following theorems.  Explain and justify any assertions made.

Problem 1:
Define a sequence recursively by: \( a_1 = 1 \) and \( a_2 = 2 \) and \( a_n = a_{n-1} + 2a_{n-2} \) for all \( n \geq 2 \)

a) Find \( a_3, a_4, a_5, a_6 \).

b) Find a formula \( f(n) \) for the \( n \)th term \( (a_n = f(n)) \).

c) Prove that \( a_n = f(n) \) for all natural numbers.

Problem 2:  Theorem: Given the Fibonacci sequence \( f_1 = 1, f_2 = 1, f_n = f_{n-1} + f_{n-2} \) for \( n > 2 \), \( f_2 + f_4 + f_6 + \ldots + f_{2n} = f_{2n+1} - 1 \), \( \forall n \in \mathbb{N} \)

Problem 3:
Theorem: If \( a_1 = a_2 = 1, \ a_n = 2a_{n-1} + a_{n-2}, \ n > 2, \) then \( a_n < 6a_{n-2} \) \( \forall n \in \mathbb{N} \) and \( n > 4 \).

Problem 4:  Given the recursive sequence: \( a_1 = 1, a_2 = 1 \) and \( a_n = 2a_{n-1} + 3a_{n-2}, n \geq 3 \),

 Show that \( a_n < 3^n \) for all \( n \in \mathbb{N} \)

Problem 5:  Theorem a jigsaw puzzle that has \( n \) pieces can be completed in using \( n-1 \) fits. (A fit means that one more piece is added to the pieces already assembled at a given point in time.)