Math 2534  Homework 7

Prove the following theorems using PMI. The write up needs to be complete using sentences to explain work and conclusions.

**Theorem 1:** \( \forall n \in N, \ 1 + 2 + 2^2 + \ldots + 2^{n-1} = 2^n - 1 \)

**Theorem 2:** \( \forall n \in N, \ 5^n - 1 \) is divisible by 4

**Theorem 3:** \( \forall n \in N \geq ?, \ (n+1)! > 2^{n+3} \)

(first determine the smallest value for which this theorem would be true.)

**Theorem 4:** \( \forall n \in N \geq ?, \ 2n + 3 \leq 2^n \)

(first determine the smallest value for which this theorem would be true.)

**Theorem 5:** for all natural numbers. If \( f(x) = \ln x \), then the \( n \)-th derivative \( f^{(n)}(x) = \frac{(-1)^{n-1}(n-1)!}{x^n} \)

(Remember that \( 0! = 1 \))