I  
I was expecting intuitive responses here only.
1) a) yes  b) R+  c) yes, since no two inputs share the same output  
d) yes, this function in onto R+ (0 not included) e) yes  
2) a) no, since f(⊕)=1 and f(⊕)=2  b) co-domain is Y  
c) not one to one d) yes e) no  
3) a) yes  b) [3, ∞)  c) yes  d) yes  e) yes  
4) a) yes  b) RxR (the Cartesian plane)  c) yes  d) yes  e) yes

II  
a) Notice that (f o g):B → B and G(f o g) = G(f(g(b)))={(1,4),(3,1),(4,6),(6,3)}  
b) Notice the (f/g)(x) could only be defined on the set {1,3,4} only.  
G(f/g) = { (1,3/4), (3, 3/2) } notice that (f/g)(4) is undefined so the domain will  
be on the set {1,3} only.  
c) No, since f(x) maps A to B and f(1) = 3 and f(3) = 3. two different inputs map  
to the same output.  
d) Yes, since for every element b in B there exist an element a in A such that  
f(a) = b.  
e) Notice that (f o g):B → B so it is one to one and onto.  
d) The G(g⁻¹) = {(4,1),(2,3),(0,4),(3,6)}. Notice that g⁻¹:A → B and can only be a  
function if the domain is defined to be A-{1}.

III  
1) (f o g)(x) = (√x - 4)² - 9 = 3x - 13, x ∈ R  
2) (g o f)(x) = √(x² - 9) - 4 = √3x² - 31, x ≥ √31/3  
3) g⁻¹(x) = x² + 4/3 = 3x² + 4/3, x ∈ R  
4) (h o f o g)(x) = h(f(h(x))) = 15x - 69, x ∈ R

IV  
a) Show that f(x) is one to one by showing that if f(a) =f(b) then a = b where a is  
in the domain and b is in the range of f(x)  
Note that f(x) is defined on the domain R-{1}

If f(a) = f(b)

\[
\frac{3a}{a+1} = \frac{3b}{b+1}
\]

3a(b+1) = 3b(a+1)

3ab + 3a = 3ba + 3b

3a = 3b

a = b
b) To show that \( f(x) \) is onto we must show that for each \( b \) in the co-domain there is exist an \( a \) in the domain \( \mathbb{R} - \{-1\} \) such that \( f(a) = b \).

\[
\begin{align*}
  b &= f(a) = \frac{3a}{a+1} \\
  b(a + 1) &= 3a \\
  ba + 1 &= 3a \\
  1 &= 3a - ba \\
  1 &= a(3-b) \\
  \frac{1}{3-b} &= a
\end{align*}
\]

There exist \( a \) in the domain for every \( b \) in the co-domain except \( b = 3 \). Therefore \( f(x) \) is not onto. If we change the domain to be \( \mathbb{R} - \{3\} \) then the inverse function will exist and it will be \( f^{-1}(x) = \frac{1}{3-x} \) \( f^{-1} : \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{-1\} \).