Solutions to Homework sheet

1)  a) \( AXBXC = \{ (a,x,1), (a,x,2), (a,y,1), (a,y,2), (b,x,1), (b,x,2), (b,y,1), (b,y,2), \\
(c,x,1), (c,x,2), (c,y,1), (c,y,2) \} \)

b) \( CXAXB = \{(1,a,x), (2,a,x), \text{etc.} \ldots \} \)

c) \( CXC = \{ (1,1), (1,2), (2,1), (2,2) \} \)

2) Not equal since the points are ordered. \((x,y)\) is not equal to \((y,x)\)

3)  a) \( P(B) = \{ \emptyset, \{a,b,c,d\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \\
\{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d,\{b,c,d\} \} \}

b) \( P(A) = \{ \emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset,\{\emptyset\}\} \}

4) \( S = \{y\} \)

5) The symmetric difference is \( A \oplus B = (A \cup B) - (A \cap B) \)
\( A \cup B = \{a,b,c,d,e,f,g\} \)
so \( A \cap B = \{a,c\} \)
\((A \cup B) - (A \cap B) = \{b,d,e,f,g\}\)

6) I will prove that \((A \cup B)^c = A^c \cap B^c\) and leave the other for you to prove.

   Proof by elements:
   \[ x \in (A \cup B)^c \rightarrow x \notin A \cup B \rightarrow x \notin A \wedge x \notin B \rightarrow x \in A^c \wedge x \in B^c \]
   \[ \rightarrow x \in A^c \cap B^c \]

Since \( x \) is an arbitrary element in \((A \cup B)^c\) the hypothesis is true for all \( x \) in \((A \cup B)^c\)

7) Prove that \((A - B) \cup (B - A) \equiv (A \cup B) - (A \cap B)\)

\[(A - B) \cup (B - A) = \]
\[(A \cap B^c) \cup (B \cap A^c) = \]
\([(A \cap B^c) \cup B] \cap [(A \cap B^c) \cup A^c] = \]
\([(A \cup B) \cap (B^c \cup B)] \cap [(A \cup A^c) \cap (B^c \cup A^c)] = \]
\([(A \cup B) \cap U] \cap [U \cap (B^c \cup A^c)] = \]
\((A \cup B) \cap (B^c \cup A^c) = \]
\((A \cup B) \cap (B \cap A)^c = \]
\((A \cup B) - (A \cap B)\)
8) Diagrams were shown in class

9) Prove that \((AXC) \cap (BXD) = (A \cap B)X(C \cap D)\)

\[(x, y) \in (AXC) \cap (BXD) \iff [(x, y) \in (AXC)] \land [(x, y) \in (BXD)] \iff\]

Consider \(x \in A \land y \in C \land \) \(\neg x \in B \land \neg y \in D\) \(\iff (x \in A \land x \in B) \land (y \in C \land y \in D) \iff\)

\[x \in (A \cap B) \land y \in (C \cap D) \iff (x, y) \in (A \cap B)X(C \cap D)\]

Therefore the Hypothesis is true.

10) Let \(G\) be the set of all good students
Let \(A\) be the set of all students that attend class (are you in this set??)
Let \(B\) be the set of all bald headed people
Let \(D\) be the set of all people who receive a degree.

\[G \subseteq A\]
\[B \subseteq D\]
\[G^c \subseteq D^c \text{ Therefore } D \subseteq G\]

\[\text{so } B \subseteq D \subseteq G \subseteq A\]

Therefore all baldheaded people attend class