Selected Solutions for in class Lecture sheet on PMI-1

The Correct write up will include complete sentences and the following outline:

1) Statement of the Theorem
2) Verification of base cases (Elements in the truth set)
3) Assume true up to some arbitrary natural number k.
4) State intent to prove true for the k+1 term. Be sure to show what the results should be.
5) Body of proof: Justify each step.
6) Statement of conclusion: Since I have assumed true up to k and proved true for k+1, my hypothesis is true for all natural numbers for n > ??.

3) Theorem: \( 2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2, \quad \forall n \in \mathbb{N} \)

Using the method of the principle of math induction, I will first verify that the hypothesis is true for at least one value of n. Consider n=1, then

\[
2 = 2^{1+1} - 2
\]

2 = 2

We will also consider n = 2 to get

\[
2 + 2^2 = 2^{2+1} - 2
\]

6 = 6

I have now shown that the truth set is \{1, 2\}

Now assume the hypothesis is true up to some arbitrary natural value k: \( 2 + 2^2 + 2^3 + \ldots + 2^k = 2^{k+1} - 2 \)

I will now prove true for k+1. I will show that \( 2 + 2^2 + 2^3 + \ldots + 2^{k+1} = 2^{k+2} - 2 \)

Now I will consider the k+1 term

\[
2 + 2^2 + 2^3 + \ldots + 2^k + 2^{k+1} =
\]

by inductive assumption we have

\[
(2^{k+1} - 2) + 2^{k+1} =
\]

\[
2(2^{k+1}) - 2 =
\]

\[
2^{k+2} - 2
\]

Since I have assumed true for k and proved true for k+1, the hypothesis is true for all natural numbers.
5) Theorem: $1 + 2^n < 3^n$, $\forall n \in \mathbb{N}$, $n \geq 2$

Using the method of the principle of math induction, I will first verify that the hypothesis is true for at least one value of $n$. Consider $n=2$, then

$1 + 2^2 < 3^2$

$5 < 9$

I have now shown that the truth set is $\{2\}$

Now assume the hypothesis is true up to some arbitrary natural value $k$: $1 + 2^k < 3^k$

I will now prove true for $k+1$. I will show that $1 + 2^{k+1} < 3^{k+1}$

Now I will consider the $k+1$ term

$1 + 2^{k+1} = 1 + 2 \cdot 2^k = 1 + 2^k + 2^k$

by inductive assumption we have that

$(1 + 2^k) + 2^k < 3^k + 2^k < 3^k + 3^k$ since $n \geq 2$

so $3^k + 3^k = 2(3^k) < 3(3^k) = 3^{k+1}$

Since I have assumed true for $k$ and proved true for $k+1$, the hypothesis is true for all natural numbers and $n \geq 2$.