How to Reduce a Matrix

I. **Use Elementary Row Operations for Matrices:**

1. You can interchange any two rows of a matrix.
2. You can multiply any row in a matrix by a nonzero scalar (real number).
3. You can add any two rows together and replace either equation with the results.

Special Rule:

4. You can add the multiple of one row to another row and replace the row that was not multiplied with the results.

II. **Notation:**

1. Interchange any two rows. The symbol $R_i \leftrightarrow R_j$ means "interchange the i^{th} and j^{th} rows."
2. Multiply any row by a nonzero constant. The symbol $kR_i \rightarrow R_i$ means "the i^{th} row is multiplied by the nonzero constant k to obtain a new i^{th} row."
3. Add any two rows together and replace either equation with the results. The symbol $R_i + R_j \leftrightarrow R_j$ means "add the i^{th} row and the j^{th} row together and put the results in the j^{th} row." We could also have put the results in the i^{th} row ($R_i + R_j \leftrightarrow R_i$).
4. Multiply one row by a nonzero constant, add to another row, and replace the non-multiplied row. The symbol $kR_i + R_j \rightarrow R_j$ means "multiply the i^{th} row by the nonzero constant k and add this product to the j^{th} row to obtain a new j^{th} row."

III. **Put Matrix into Reduced Row Echelon Form** (using Gauss-Jordan elimination method): A matrix is said to be in Reduced Row Echelon Form if all of the following are true:

1. The first nonzero entry of each row, called the leading entry of the row, must be 1.
2. All other entries in the same column as a leading 1 are zeros.
3. The first nonzero entry in each row must always be to the right of the first nonzero entry of each row above it.
4. Any rows that consist entirely of zeros are at the bottom of the matrix.