Answers to Proofs: Math 2534

Answers to Proof Homework sheet.

1) The sum of any even integer and any odd integer is odd.

Proof:

Let a be the even integer and b be the odd integer. By definition of even and odd we have that a = 2n and b = 2m + 1. Consider the sum a + b = 2n + 2m +1 2(n + m) +1 = 2k +1 where k = n + m is an integer. Therefore by definition of odd we have shown that a + b is odd my hypothesis is true.

2) The product of any even integer and any other integer is even.

Proof:

We need to consider two cases:

Case1: Let a be even and b be odd. Then by definition a = 2n and b = 2m + 1 for n, m integers. Now consider ab = (2n)(2m + 1) = 4nm + 2n = 2(2nm + n) + 2k where k = 2nm + n is an integer. Therefore by definition the product ab is even.

Case 2: Let both a and b be even, then a = 2n and b = 2m for n, m integers. Consider the product ab = (2n)(2m) =2(2nm) = 2k where k = 2nm is an integer. Therefore the product ab is even by definition and both cases verify the truth of my hypothesis.

3) For n a positive integer, If n > 2 and n is prime then n is odd.

Proof: (by contradiction)

Given that n > 2 is prime assume that n is even. Then the factors of n would be at least 2, n, 1. But this contradicts the definition of prime which states that a prime number can only have the factors 1, n. Therefore n can not ever be even so it must be odd.

4) The product of any two consecutive integers is even.

Proof:

If a and b are consecutive integers then if a =2n then b = 2n +1 for some integer n. Consider the product ab = (2n)(2n + 1) = $4n^2 + 2n = 2(2n^2 + n) = 2k$ for $k = 2n^2 + n$. There fore the product ab is even and the hypothesis is true.

5) If a, b are integers, The product is odd if and only if a and b are both odd.

Proof:

Part 1) If a and b are both odd then ab is odd.

Proof: By definition a = 2n + 1 and b = 2m + 1 for n, m integers. Now consider the product ab = (2n + 1)(2m + 1) = 4nm +2n +2m +1= 2(2nm +n + m) + 1 = 2k where k = 2nm + n + m is an integer. Therefore the product ab is odd by definition of odd.

Part 2) If ab is odd then a and b are each odd.

Proof: (by contradiction) Given that ab is odd assume that a and b are not both odd. If a and b are not both even then we must consider two cases.

Case 1: Let a be even and b is odd. By definition we have a = 2n and b = 2m + 1 where n, m are integers. Consider ab = (2n)(2m +1)= 4nm + 2n = 2(2nm +n) which is even. This is a contradicts that ab is in fact odd.
Case 2: Let a and b both be even. By definition a = 2n and b = 2m for n, m integers. Consider the product ab = (2n)(2m) = 2(2nm) = 2k for k = 2nm is an integer. There we have that ab is even which is a contradiction to the fact that ab is odd.
There fore by the two cases above a and b must both be odd.

By Part one and Part two above we have proven the original statement.

6) Given that a,b,c are integers, if a divides b evenly and a does not divide c evenly, prove that a does not divide (b + c) evenly.
Proof: (by contradiction)
Since a divides b, by definition b = aq_1 for some integer q_1, assume that a also divides b + c then we have b + c = aq_2 for some integer q_2. therefore c = $aq_2 - aq_1 = a(q_2 - q_1)$ = aq_3 for some integer q_3 . This states that a divides c evenly which is a contraction so a does not divide a + b as assumed.

7) This statement was a typo and should have been as follows:
given a,b,c are integers, If a does not divide bc evenly then a does not divide b.
Proof: (by contradiction)
Given that a does not divide bc assume that a does divide b. Then by definition we have that b = aq. By algebra we have that bc = aqc = a(qc) = aq_1 where q_1= qc is an integer. So a divides bc which is a contradiction and a does not divide b.

8) We have the following hypothesis: An integer n is even iff n^2 is even

Proof must be in two parts:
Part 1: If n is even, then n^2 is even.

Proof: If n is even then by definition n = 2a for some integer a. Therefore n2 = n*n = (2a)(2a) = $2(2a^2)$ which is also even by definition. Therefore I have shown that if n is even then n2 is also even.

Part2: If n^2 is even, then n is even

Proof: For this proof I will use the contrapositive statement which is equivalent to the original statement. If n is odd, then n^2 is odd.
Therefore by definition of odd we have that n = 2p + 1 for some integer p. Now consider n^2 =n*n = (2p+ 1)(2p+1) = $4p^2 + 4p + 1$ = $2(2p^2 + 2p)+1$ = 2k + 1, where k is the integer value $2p^2 + 2p$. So by the definition of odd we have that n^2 is odd. There the contrapositive is true and the original statement (which is equivalent) is also true.

Have successful proven both parts of my hypothesis, I now have that an integer n is even iff n^2 is even

9) If x, y are integers and x and y are both odd, then x + y is even.

a) Direct Proof: Since x and y are both odd, we have that $x = 2a + 1$ and $y = 2b + 1$ for a, b integers.
Consider the sum $x + y = (2a + 1) + (2b + 1) = 2a + 2b + 2 = 2(a + b + 1) = 2k$ where k is the integer $a + b + 1$. By definition we have that x + y is even and I have proven my hypothesis.

b) Proof by Contrapositive: Restate the original statement to be as follows:
If x + y is odd, then x and y are not both odd:

Consider $x + y = 2n + 1$ by definition of odd for some integer n. Then we have that $x = (2n + 1) – y$. We will now consider the possible cases for y.

Case 1: If y is odd then $y = 2a + 1$ for some integer a and the difference of two odd numbers is $(2n + 1) – (2m + 1) = 2(n+m)$ which makes x even.

Case 2: If y is even then $y = 2a$ for some integer a and the difference $x = (2n + 1) – 2a = 2(n + a) + 1$ is odd.

Therefore I have proved the contrapositive statement is valid ane the original statement is also true.

c) Proof by Contradiction: Restate the original to be that x and y are odd integers and the sum x + y is also odd.

We have that $x + y = 2n + 1$ by definition of odd for some integer n. Then we have that $x = (2n + 1) – y$. Given that y is odd then $y = 2a + 1$ for some integer a and the difference of two odd numbers is $(2n + 1) – (2m + 1) = 2(n+m)$ which makes x even.
But this contradicts that both x and y are given to be odd. Therefore the original must be true and if x, y are integers and x and y are both odd, then x + y is even.

10) Given n, a, b are integers: If $n|a$ and $n|(a+b)$, Then $n|b$

Proof:

Since $n|a$ we have that a = nq and $n|(a+b)$ will give us nk = a + b by definition of divisible. Therefore nk = nq + b which will give us nk – nq = b. Distribute out the n to get n(k-q) = nc= b where c is the integer k-q. By definition of divisible, we now have that $n|b$.

11) $\{\exists x | x \in Z \wedge x \notin P\}$ where Z is the set of all integers and P is the set of all primes.

12) $\{\forall x, x \in E \wedge x \in P\}$ where E is the set of all even integers and P is the set of perfect

squares.