Math 2534  Answers To Test 1:

1) 
\[
\sim[(p \land (p \rightarrow q)) \rightarrow q] = \sim[(p \land (\sim p \lor q)) \lor q]
\]
\[
= (p \land (\sim p \lor q)) \land \sim q = [(p \land \sim p) \lor (p \land q)] \land \sim q =
\]
\[
[F \lor (p \land q)] \land \sim q = p \land (q \land \sim q) = p \land F = F
\]

2) We have that \( D \rightarrow I \rightarrow M \rightarrow P \) where D: drinking, I: Illness, M: miss class, P: poor grades. The only thing that can be determined from Dan misses class is that he will have poor grades. Otherwise M is a necessary condition and can give us not information about Dan drinking.

3) \( p \rightarrow q \) where \( p \) is the statement: it is fall and \( q \) is the statement VA Tech plays football.

1) \( p \rightarrow q \) equivalent
2) \( p \rightarrow q \) equivalent
3) \( q \rightarrow p \) not equivalent
4) \( p \rightarrow q \) equivalent

4) \( p \rightarrow (q \rightarrow k) \) is true and \( (q \rightarrow k) \) is false. Therefore in order for \( p \rightarrow (q \rightarrow k) \) to be true \( p \) must be false and in order for \( (q \rightarrow k) \) to be false \( q \) must be true and \( k \) false.

5) If 8 does not divide \( n^2 \) then 8 does not divide \( n \).

Proof using the contrapositive: If \( 8 \mid n \rightarrow 8 \mid n^2 \)

Since 8 divides \( n \) we have that \( n = 8q \) for some integer \( q \). This will give us the following: \( n^2 = n^2 = 8q^2 \). Therefore by the definition of divisible we have \( n^2 \) divisible by 8. Since the contrapositive is true, it’s equivalent, the original statement is also true.

6) a) \( \{ x \in S | D(x) \} \) S stands for the shelter and \( D(x) \) is the statement \( x \) is a dog.
   b) The truth set contains \{2 yellow dogs, 5 brown dogs, 1 black dog\}
   c) \( \sim \{ x \in S | D(x) \} = \{ \forall x \in S, \sim D(x) \} \) There are no dogs in this shelter
   d) Since this is a false statement, there are not truth values.

7) a) Every team can be identified by its uniform colors (True)
   b) There is one set of uniform colors that identifies all teams. (False)
8) If $x$ and $y$ are rational numbers and $k$ is irrational, then $x + yk$ is irrational.

Proof by contradiction: Assume that $x + yk$ is rational. Therefore we have that $x + yk = a/b$ where $a, b$ are integers.

\[
x + yk = a/b
\]

\[
yk = a/b - x
\]

\[
k = \frac{(a/b) - x}{y}
\]

the difference of rational numbers are rational

the quotient of rational numbers is rational

Therefore $k$ is rational

But this contradicts the hypothesis that $k$ is irrational. Therefore the original statement is true and $x + yk$ is irrational.

9) If $x = m \mod d$ and $y = n \mod d$ then $x + y = (m + n) \mod d$.

Proof: Given that $x = m \mod d$ and $y = n \mod d$, we have by the definition of Quotient Remainder Theorem that $m = dq + x$ and $n = dp + y$ where $q$ and $p$ are integers.

\[
m + n = dq + x + dp + y = d(q + p) + (x + y) = dk + (x + y)
\]

By definition of Quotient Remainder Theorem we now have that $(m + n) \mod d = (x + y)$