Math 2534 Solutions to HW 8

1) Theorem: The product of an even integer and an odd integer is even.

Proof:
Let a be even and b be odd. Then by definition a = 2n and b = 2m + 1 for n, m integers. Now consider ab = (2n)(2m + 1) = 4nm + 2n = 2(2nm + n) + 2k where k = 2nm + n is an integer. Therefore by definition the product ab is even.

2) Theorem: The sum of any odd integer and any even integer is odd.

Proof:
Let a be the even integer and b be the odd integer. By definition of even and odd we have that a = 2n and b = 2m + 1. Consider the sum a + b = 2n + 2m +1 = 2(n + m) +1 = 2k +1 where k = n + m is an integer. Therefore by definition of odd we have shown that a + b is odd and my hypothesis is true.

3) Theorem: The difference of any odd integer minus any even integer is odd.

Proof:
Let a be the even integer and b be the odd integer. By definition of even and odd we have that a = 2n and b = 2m + 1. Consider the difference b-a = 2m +1 – 2n = 2m-2n +1 = 2(m-n) +1 where k = m-n is an integer. Therefore by definition of odd we have shown that b-a is odd and the hypothesis is true.

4) Theorem: The difference of any two rational integers is rational.

Proof:
Let r and q be two rational numbers. Then by definition of rational 
\[ r = \frac{a}{b}, q = \frac{c}{d} \]
where a,b,c,d are integers and b and d are not zero.
Consider 
\[ r - q = \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} = \frac{m}{n} \]
where m = ad-bc and n = db and m and n are integers. Therefore by definition of rational we have that r – q is also rational.

5) If the sum a + b is even and a – c is odd where c is even. Determine if the sum of (a + b) + (a – c) is odd or even. Can you also determine if each a and b are odd or even.

By previous theorems you can determine that (a+b) + (a-c) is odd and both a and b are also odd. I leave the discussion write up to you.