Math 2534 Solutions to HW 8

Problem 1:
\[ A = \{a, b, \{a, b\}, c\} \text{ and } B = \{a, b\} \]

a) \( A \cup B = A \)
b) \( A - B = \{\{a, b\}, c\} \)
c) \( T \)

d) \( T \)

e) \( P(B) = \{\emptyset, \{a, b\}, \{a\}, \{b\}\} \)
f) \( A \oplus B = (A - B) \cup (B - A) = \{\{a, b\}, c\} \cup \emptyset = \{\{a, b\}, c\} \)

Problem 2: See Class Notes

Problem 3: Let \( D \) be all ducks, \( S \) be all things that swim, \( C \) be all cats, \( L \) be all things that climb

a) \( D \subseteq S \)
b) \( C \subseteq L \)

c) \( D^C \subset L^C \equiv L \subset D \)

Therefore \( C \subset L \subset D \subset S \)
Which says that all cats swim.

Problem 4: TH: For any sets \( A, B, \) and \( C, (A \cup B) \cup C = A \cup (B \cup C) \)

Proof:
\[ \forall x \in (A \cup B) \cup C \rightarrow x \in (A \cup B) \lor x \in C \text{ by definition of Union} \]
\[ \rightarrow (x \in A \lor x \in B) \lor x \in C \text{ by definition of Union} \]
\[ \rightarrow x \in A \lor (x \in B \lor x \in C) \text{ by logic associativity} \]
\[ \rightarrow x \in A \lor (x \in (B \cup C)) \text{ by definition of union} \]
\[ \rightarrow x \in A \cup (B \cup C) \text{ by definition of union} \]

Therefore we have that \( (A \cup B) \cup C \subseteq A \cup (B \cup C) \)

By reverse steps we can show that \( A \cup (B \cup C) \subseteq (A \cup B) \cup C \)

So by definition of equal sets we have that \( (A \cup B) \cup C = A \cup (B \cup C) \)