Math 2534 Solution to Homework 7 Sequences and PMI

Problem 1: Verify that $\sum_{i=1}^{8} i = \frac{n(n+1)}{2}$ where $n = 8$

\[1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = \frac{8(8+1)}{2}\]

\[36 = \frac{72}{2}\]

Problem 2: Given the recursive sequence $a_1 = 4, \quad a_n = a_{n-1} + 3$, find the sequence function that will give the same results.

$F(n) = 3n + 1$

Problem 3: If you are given a sequence function $f(n) = 4^{n-1}$, find the recursive sequence that will give you the same results.

$a_1 = 1, \quad a_n = 4a_{n-1} \quad n > 1$

Problem 4: Reduce $\frac{(n+1)!}{(n-2)!}$ to get $(n-1)(n)(n+1)$

Problem 5: Evaluate the sequence $1(1!) + 2(2!) + \ldots + n(n!)$ for $n = 5$. ANS is 719
Problem 6: Use PMI to prove the following:

Theorem: For all natural numbers, \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \)

Proof is by PMI:
I will verify that the hypothesis is true for at least one value of \( n \).

Consider \( n = 1 \) to get \( 1 = \frac{1(1+1)}{2} \) which gives us that \( 1 = 1 \)

Also consider \( n = 2 \) to get that \( 1+2 = \frac{2(2+1)}{2} \) which gives us that \( 3 = 3 \)

I will now assume that the hypothesis is true up to some arbitrary value \( k \in \mathbb{N} \), i.e.: that
\[ 1+2+3+\ldots+k = \frac{k(k+1)}{2} \]

I will prove that the hypothesis is also true for \( k+1 \). i.e.: \( 1+2+3+\ldots+(k+1) = \frac{(k+1)(k+2)}{2} \)

Proof:
Consider the \( k+1 \) term which is \( 1+2+3+\ldots+k+(k+1) \) and notice that by the inductive assumption from above, we already have that \( 1+2+3+\ldots+k = \frac{k(k+1)}{2} \) so I will replace the first \( k \) terms with this equivalent form to get
\[ (1+2+3+\ldots+k) + (k+1) = \frac{k(k+1)}{2} + (k+1) \]

And by doing some algebra, we will have:
\[ (1+2+3+\ldots+k) + (k+1) = \frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2} \]

Therefore, since I have assumed true up to \( k \) and proved true for \( k+1 \), the hypothesis is true for all natural numbers.